The Product-variety model:

The Product-Variety Model, introduced by Paul Romer in his 1990 paper *"Endogenous Technological Change"*, builds on the Solow (1956) model but incorporates endogenous technological change. Unlike earlier models where technological progress was assumed to occur at a constant external rate, here it is driven by the choices of agents within the economy. These agents have market incentives to invest in innovation, making technological progress a key driver of sustained capital accumulation and long-term economic growth.

Preliminaries:

Innovation:

There are many ways to model innovation in Economics, from which we can identify two broad families. The French economist Philippe Aghion for instance focuses mostly on *"vertical innovation"*, where there is a constant set of goods, and these goods become more productive through innovation. Thus innovation leads to growth because the goods existing in the economy become more and more productive. The Romer (1990) model we will now study, however, emphasizes *"horizontal innovation"*, where innovation leads to the creation of new varieties of intermediate goods (the number of intermediate goods necessary for production, A, increases). This increase in varieties/the number of different machine types enhances the productivity of unskilled labor in the final good sector, leading to higher overall output. Obviously, one could also model innovation as a mixture of both vertical and horizontal dynamics.

Competition:

The two most used market competition types used in economics and the two most extreme cases of competition are perfect competition on the one hand, and monopoly on the other hand.

Perfect Competition: In perfect competition, you have a lot of firms in the same market, and because many firms share the same market, all firms are pretty small. Because of their small size, firms have no market power at all. This means that firms have no power to set prices as they want, for them, the price is as given, in the sense that they have no influence at all on it. As consequence, when we will try to maximize the profit of a firm operating in a market under perfect competition, we will treat the price as given/as an exogenous parameter from the firm's perspective. It turns also out that under perfect competition, firms earn no profit at all. This is because of what we call the free-entry condition. The free-entry condition means that when firms in a perfectly competitive market earn positive profits, new firms are attracted to the market by this profit. As more firms enter, the increased competition reduces the profits available to each firm, since the same market demand is now split among more producers. This process continues until profits are driven down to zero, at which point no more firms have an incentive to enter.

Monopoly: Monopoly describes the case in which there is only one firm in a market. Because the firm is alone on a market, it has a huge market power. This means that the firm can influence highly the price of the good it is selling. The firm knows that if it wants to sell more goods, it will have to sell it at a lower price. As a consequence, when we will maximize the profit of the monopolistic firm, we will replace the price in profit by the demand function, to model the idea that this firm can influence the market price of the good it is selling.

Description of the Model:

This model has two main sides: the consumption side and the production side.

Consumption Sector

Households in this economy consume a final good, denoted by Y_t, and supply labor in two forms:

- **Unskilled Labor (N_Y):** These workers are employed in the production of the final good and earn a wage w.
- **Skilled Labor ($N_{R\&D}$):** These workers, also called "scientists," are employed in the Research and Development (R&D) department of the intermediate good sector, earning a wage r_A. Their role is to innovate and create new varieties of capital goods.

Production Sector

The production side is structured into three interconnected layers:

1. **Final Goods Sector:**

$$Y_t = N_Y^{1-\alpha} \int_0^A x_i^\alpha \, di$$

The final good, Y_t, is produced using unskilled labor, N_Y and a continuum/variety of intermediate goods/"machines", x_i. Unlike earlier models where capital was a single variable, K, this model assumes a more realistic scenario where different types of "machines" or capital goods (x_i) exist and are assumed to be different. The total number of these capital types are given by A. Finally note that here we use an integral instead of a sum to model that in reality, there can be many (possibly infinitely many) varieties that contribute to production in very small ways.

As mentioned above, these different capital types are not perfect substitutes for one another - some may be more complementary (like computers and communication networks), while others might substitute each other (like trucks and trains). This formulation captures the diversity of capital goods and their contributions to production.

Importantly, the final good market is assumed to be perfectly competitive, meaning that no single firm has the power to set prices. Instead, firms earn zero profit in equilibrium, which comes from the free-entry condition of competitive markets.

2. **Intermediate Goods Sector:** The production of the A types of "machines" occurs in the intermediate goods sector. Each machine type, x_i, is produced by a firm that uses two inputs: capital and the work of one scientist (skilled labor).

We make the simplifying assumption, that capital used to produce "machines" are units of the final good. More precisely, after households consume a portion of the final good, the remaining part, $Y_t - C_t$ is invested as capital to produce the intermediate goods.

Unlike the final goods sector, the intermediate goods market is characterized by **monopolistic competition**, where each intermediate input x_i is produced by a single firm with monopoly power. Monopolistic competition allows firms to maintain exclusivity over their new products, earn profits, and thereby have an incentive to invest in innovation.

3. **Research and Development (R&D) sector:** The R&D sector is responsible for creating new varieties of intermediate goods and selling them to the intermediate goods sector. There are many ways to model how new varieties are created, in practice we will assume a simple law of motion for A, the number of varieties in the economy:

$$\frac{A_{t+dt} - A_t}{dt} = \frac{dA_t}{dt} = \lambda N_{\text{R\&D}}$$

where $\lambda > 0$ is the productivity of R&D labor and $N_{\text{R\&D}}$ is the number of skilled workers (scientists) employed in R&D. This formulation captures the idea that the rate of technological progress (increase in varieties) depends on the resources devoted to innovation. Each time a new variety is created by the R&D sector, it will sell this new technology at a price r_A to a new firm which will specialize in that new design of machine. In practice, we could think of this price r_A either like a patent (i.e. the monopolist firm buys this new technology to the R&D sector) or that each time a new variety is created, the new entering firm needs to hire a scientist at a wage r_A in order to implement this new technology properly.

Key Features of the Model:

Interconnections between Sectors:

This model is complex because every blocks of the economy are interdependent. Households supply labor both for the final output sector and the R&D sector, influencing respectively current production and the pace of technological progress. The final output has a double purpose, it is consumed by households and invested as capital in the production of intermediate goods. Innovations from the R&D sector increase the number of variety of intermediate goods, which in turn increases the productivity of the final goods sector.

Conclusion:

By endogenizing technological change, this model explains sustained economic growth without relying on exogenous factors. The economy can experience perpetual growth driven by continuous innovation and capital accumulation.

From a policy point of view, this model highlights the importance of policies that encourage investment in human capital and innovation, such as education, intellectual property rights, and subsidies for innovation, as there policies can have significant effects on long-term economic growth.

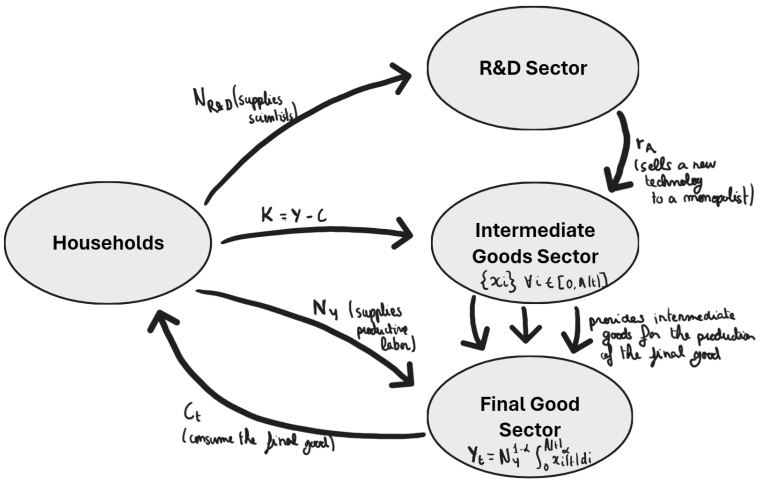

Figure: Simplified description of the Product-variety model: