A simple version of the Myopic Schumpeterian Model:

In this chapter, we develop an alternative variety-good model of endogenous growth, in which technological progress is vertical this time, that is the number of varieties in the economy remains constant, but the quality of these varieties improve as time passes.

The Model:

There is a unique final good in the economy, Y_t, which is used for consumption C_t, the production of a unique intermediate good X_t, and investment in R& D, R_t. Therefore the resource constraint of this economy is simply:

$$Y_t = C_t + X_t + R_t$$

The production technology:

We assume that labor, L, is constant. The final good is produced by perfectly competitive firms using two inputs, labor and a single intermediate product, according to the Cobb-Douglas production function:

$$Y_t = (A_t L_t)^{1-\alpha} x_t^{\alpha}$$

where Y_t is output of final good in period t, A_t is a parameter that reflects the productivity of the intermediate input that period, and x_t is the amount of intermediate input used. We normalize the price of the final output to one.

The intermediate product is produced by a monopolist each period, that uses as an input, the final good in a one to on relationship. That is the production function is simply:

$$x_t = X_t$$

That is, for each unit of intermediate product, the monopolist must use one unit of final good as input.

Innovation:

Growth results from innovations that raise the productivity parameter A_t, which improves the quality of the intermediate product. Each period there is one person (the "entrepreneur") who has an opportunity to attempt an innovation. If she succeeds, the innovation will create a new version of the intermediate product, which is more productive than previous versions. If she fails, then a new entrepreneur will take the lead.

Let us denote last period's productivity as A_{t-1}. Specifically, the productivity of the intermediate good in use will go from last period's value A_{t-1} up to $A_t = \gamma A_{t-1}$, where $\gamma \geq 1$. This happens with a productivity z. On the other hand, if she fails, at a probability $1 - z$, then there will be no innovation at t and, in this case, another randomly chosen monopolist will produce the intermediate good with the old productivity that was used in $t - 1$, so $A_t = A_{t-1}$. Hence

$$A_t = \begin{cases} \gamma A_{t-1} & \text{if entrepreneur is successful,} \\ A_{t-1} & \text{if entrepreneur fails.} \end{cases}$$

In order to innovate, the entrepreneur must conduct research, a costly activity that uses the final good as its only input. As indicated above, research is uncertain, for it may fail to generate any innovation. But the more the entrepreneur spends on research, on z, the more likely she innovates. Specifically, we assume that to innovate from A_{t-1} up to $A_t = \gamma A_{t-1}$ with probability z_t, one must spend the amount

$$R_t = \frac{\delta z_t^2}{2} A_{t-1}$$

final good on research, where δ is a parameter which inversely measures the productivity of the research sector. The cost is an increasing function of δ, of the probability of success for the entrepreneur z_t and of the current technological level A_{t-1} (the more productive you are, the costlier it becomes to be even more productive).

Timing of events

Now we can summarize the timing of events in this model:

- **step 0**: Period t begins with the initial productivity A_{t-1},
- **step 1**: a randomly chosen entrepreneur invests in R&D by choosing (z_t, R_t),
- **step 2**: innovation (success/failure) is realized, and productivity evolves (either to γA_{t-1} or remains at A_{t-1}),
- **step 3**: production of the intermediate good (x_t) takes place,
- **step 4**: production of the final good (Y_t) takes place,
- **step 5**: consumption (C_t) takes place, and period t ends.

Solving the model:

There exists two kinds of solution to these models, each delivering a different outcome and interpretation of the model. First, there is the Competitive Equilibrium. We get the solution to the Competitive Equilibrium by allowing each agent to maximize its own payoff. Second, there is the Central Planner solution, in which we assume the existence of a central entity. This Central Planner knows everything, can decide for all agents in the economy, and its only purpose is to maximize the payoff of society as a whole, not of individual agents, this can for instance be to maximize consumption or total utility of consumers.

Solving the Competitive Equilibrium:

The model is solved by backward induction, this means that we start from the last step and we go progressively to the first one. In each period, we first compute the equilibrium production and profit of the final good producer, then of a successful innovator, we move back one step and compute the optimal innovation intensity by the firm selected to be an innovator.

Equilibrium production and profits:

We start from step 4. The final good producer maximizes the following objective function

$$\max_{x_t, L_t} \left\{ (A_t L)^{1-\alpha} x_t^{\alpha} - w_t L_t - p_t x_t \right\}.$$

Therefore we can express the inverse demand for intermediate good x_t and labor as

$$p_t = \frac{\partial Y_t}{\partial x_t} = \alpha (A_t L)^{1-\alpha} x_t^{\alpha - 1}$$

and

$$w_t = (1-\alpha) A_t^{1-\alpha} L^{-\alpha} x_t^{\alpha},$$

where we already imposed that $L_t = L$.

Now we move to step 3. At step 3 A_t is known, as the realization of stochastic innovation process occured earlier at step 2. Remember that this market is intermediate machines is in monopoly, so the each firm producing one machine has a huge market power. Because of the monopoly, the monopolist with productivity A_t, taking the demand for the intermediate good as given, and maximizes her profit $\Pi(A_t)$, measured in units of the final good:

$$\Pi(A_t) = \max_{x_t, p_t} \{p_t x_t - x_t\}$$
$$\text{such that } p_t = \alpha (A_t L)^{1-\alpha} x_t^{\alpha - 1}$$

Here, p_t is the price of the intermediate product relative to the final good. That is, her revenue is price times quantity $p_t x_t$ and her cost is her input of final good, which must equal her output x_t. Substituting the constraint into the objective function we get

$$\Pi(A_t) = \max_{x_t} \left\{ \alpha (A_t L)^{1-\alpha} x_t^{\alpha} - x_t \right\}.$$

which implies when computing the FOC that the equilibrium quantity for the intermediate good is:

$$x_t = \alpha^{\frac{2}{1-\alpha}} A_t L$$

and the equilibrium price is:

$$p_t = \alpha (A_t L)^{1-\alpha} (\underbrace{\alpha^{\frac{2}{1-\alpha}} A_t L}_{x_t})^{\alpha - 1} = \frac{1}{\alpha} = p$$

Note that the equilibrium price is above the marginal cost (which is equal to one) since $\alpha \in (0, 1)$. What firms earn after having payed the marginal cost is called the price markup, which is governed by α. As α goes to one, the markup vanishes and the price becomes equal to the marginal cost.

Finally, the equilibrium profit, that is the profit once we plug the optimal quantity and price for the intermediate good, gives:

$$\Pi^*(A_t) = p^* x_t^* - x_t^* = A_t L \left(\alpha^{\frac{2}{1-\alpha}} \frac{1}{\alpha} - \alpha^{\frac{2}{1-\alpha}} \right) = A_t L (1 - \alpha) \alpha^{\frac{1+\alpha}{1-\alpha}}$$

Substituting, the optimal quantity of intermediate good in the production function of the final output, we get:

$$Y_t = (A_t L)^{1-\alpha} (y_t^*)^{\alpha} = \alpha^{\frac{2\alpha}{1-\alpha}} A_t L$$

Equilibrium innovation intensity:

In step 2, for any given innovation rate z_t,

$$A_t = \begin{cases} \gamma A_{t-1} & \text{with probability } z_t, \\ A_{t-1} & \text{with probability } 1 - z_t. \end{cases}$$

We now move back to step 1 and consider the innovation investment decision of the entrepreneur who has the opportunity to innovate at date t. If the entrepreneur at t successfully innovates, she will become the intermediate monopolist that period, because she will be able to produce a better product than anyone else. Otherwise, the monopoly will pass to someone else chosen at random, who is able to produce last period's product. Thus the entrepreneur will choose the innovation intensity z_t to maximize. Thus to infer the optimal R&D search intensity z_t, we maximize the expected profit of the entrepreneur, which looses everything in case of failure.

$$\max_{z_t} \{z_t \Pi[\gamma A_{t-1}] + (1 - z_t) \times 0 - \frac{\delta z_t^2}{2} A_{t-1}\}$$

or equivalently to

$$\max_{z_t}\{z_t(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\gamma A_{t-1}L - \frac{\delta z_t^2}{2}A_{t-1}\},$$

The first-order condition is simply:

$$\delta z_t = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\gamma L,$$

which, yields the equilibrium innovation intensity:

$$z_t^* = \frac{\gamma L}{\delta}(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} = z^*$$

As a consequence, with probability z^*, A_{t-1} goes to γA_{t-1}, that the growth rate of capital is $(\gamma-1) > 0$. On the contrary, with probability $(1-z^*)$ capital does not grow. Given $Y_t = \alpha^{\frac{2\alpha}{1-\alpha}}A_tL$, we have that the economy's average growth rate equals the frequency of innovations (z^*) times the size of innovations $(\gamma-1)$:

$$\mathbb{E}\left(\frac{Y_{t+1}-Y_t}{Y_t}\right) = z^*(\gamma-1) = \frac{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\gamma L}{\delta}(\gamma-1)$$

We thus see clearly that growth increases with the productivity of innovations, measured by $\frac{1}{\delta}$. This result points to the importance of education, and particularly higher education, as a growth-enhancing device. Growth also increases with the size of innovations, measures by γ. And finally, an increase in the size of population should also bring an increase in growth by raising the supply of labor L.

We have just solved the competitive equilibrium, that is the case in which the final producer, the intermediate good producers and the entrepreneur each do what is privately good for them.

Solving the Central Planner Problem:

After having solved the problems of each economic agent, based on what is optimal for them to do, we will ask the question: What is the socially optimal level of production and R&D investment for the whole economy ?

The goal of the social planner will be to maximize utility of households, which is equivalent to maximizing the level of consumption. Again, we will use a backward induction argument. In step 4, the social planner maximizes consumption, subject to the final good and the intermediate good production technologies:

$$C_t = Y_t - X_t - R_t$$

Note that in step 5, the R&D investment is already made, hence R_t is taken as constant at this stage. Let's denote $\hat{C}(A_t, R_t)$, the maximum level of consumption, given productivity A_t and investment in R&D R_t:

$$\hat{C}(A_t, R_t) = \max_{x_t}\left\{(A_tL_t)^{1-\alpha}x_t^{\alpha} - x_t - R_t\right\}$$

From this, computing the FOC, we get that the socially optimal level of intermediate production is:

$$x_t^{SP} = A_tL\alpha^{\frac{1}{1-\alpha}} > x_t^* = \alpha^{\frac{2}{1-\alpha}}A_tL$$

and the resulting maximum consumption is:

$$\hat{C}(A_t, R_t) = (A_tL)^{1-\alpha}\left(A_tL\alpha^{\frac{1}{1-\alpha}}\right)^{\alpha} - A_tL\alpha^{\frac{1}{1-\alpha}} - R_t = A_tL\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha) - R_t$$

and finally, given $Y_t = (A_tL)^{1-\alpha}x_t^{\alpha}$, we get that:

$$Y_t^{SP} = A_tL\alpha^{\frac{\alpha}{1-\alpha}} > Y_t^* = \alpha^{\frac{2\alpha}{1-\alpha}}A_tL$$

So the decentralized economy under-produces output with respect to the social planner, and this comes from the "monopoly distortion".

Now we go one step back in the social planner's problem and we will compute the optimal innovation decision. For this we maximize expected consumption:

$$\mathbb{E}(C_t^{SP}) = \max_{z_t} \left\{ z_t \gamma A_{t-1} L \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) + (1-z_t) A_{t-1} L \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) - \frac{\delta z_t^2}{2} A_{t-1} \right\}$$

Note here that the social planner cares about consumption if innovation is successful but also in case of failure. However, in the decentralized economy, the entrepreneur cares only about the success since in the case of a failure, the market is served by another firm.

From this we get the socially optimal innovation rate by taking the FOC:

$$z_t^{SP} = (\gamma - 1) \frac{L \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha)}{\delta}$$

Under some conditions we have that $z_t^{SP} > z_t^*$.

We have thus found two market inefficiencies:

1. Under-production coming from the monopoly rent.
2. Under-investment coming from the fact that the entrepreneur only care about their own success and are fired in case of a failure.