NASH EQUILIBRIUM:
THEORY

PREFERENCES

Players = actors

model → subway

- **Ordinal preferences** compare items, but not the intensity of preferences.
    - For example, I like bananas more than apples.
- **Cardinal preferences** compare items but, also, the intensity of preferences.
    - For example, I like bananas 2.5 times more than apples.
- However, cardinal preferences require more assumptions.
- For now (i.e. chapters 2-3), we will assume preferences are ordinal.

ORDINAL PREFERENCES

payoffs.

- If person i strictly prefers item A to item B, we write: $A \succ_i B$.

$u(A) \overset{=10}{\in} \mathbb{R}$

$u(B) \underset{=5}{\in} \mathbb{R}$

$\underset{10}{u(A)} \succ \underset{5}{u(B)}$ $>$

- If person i weakly prefers item A to item B, we write: $A \succeq_i B$

$\underset{10}{u(A)} \geq \underset{10 \text{ or } 7}{u(B)}$ ...

- If person i is indifferent between item A to item B, we write: $\boxed{A \sim_i B}$ $\iff$ $A \succeq_i B$ and $B \succeq_i A$.

$u(A)=u(B)$

RATIONAL

- We make 2 assumptions on preferences. Specifically,
  - that preferences are **complete** (each pair can be compared); that is, either $A \succeq_i B$ or $B \succeq_i A$ or both; and
  - that preferences are **transitive**; that is, if $A \succ_i B$ and $B \succ_i C$, then $A \succ_i C$.

PAYOFF FUNCTION

- When using ordinal preferences, we can assign a payoff function to the preferences.
- Example 1: if $A \succ_i B$, then, we could assign, for example,
  - $u(A) = 2$ and $u(B) = 1$.
  - In fact, any $u(A)$ and $u(B)$ such that $u(A) > u(B)$ would do.
- Example 2: if $A \succ_i B$ and $B \succ_i C$, then, we could assign, for example,
  - $u(A) = 3$, $u(B) = 2$ and $u(C) = 1$.
  - In fact, any $u(A)$, $u(B)$ and $u(C)$ such that $u(A) > u(B) > u(C)$ would do.
- Since preferences are ordinal, the payoff function does not convey intensity.

Game : introduction.

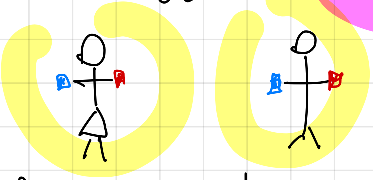
two players : Girls vs boys

player 1 : random girl

Player 2: random boy

Each player may choose between Red or Blue

Simultaneous



payoffs

colors		payoffs.	
Girl	Boy	Girl	Boy
B	B	7	3
B	R	7	4
R	B	3	4
R	R	10	5

Boy → Player 2

Player 1 → Girl

		B	R
Girl	B	7 , 3	7 , 4
	R	3 , 4	10 , 5

Strategic Games with Ordinal Preferences

Game theory : study of strategic interactions
v.s. decision theory.

Definition

A **strategic game with ordinal preferences** consists of:

1. a set of **players**,
2. a set of **actions** for each player, and
3. **preferences** over the set of action profiles for each player.

- An action profile is a list of specific actions for each player. $u_G(B,B) = 7$ $u_G(B,R) = 7$
- The game does not contain time information, as it assumes players' moves are simultaneous.

NORMAL-FORM GAME TABLE

Strategic

- A 2×2 game is represented with a game table as illustrated below.

Bimatrix

2 players: 1 and 2

Actions pl 1: up or down

Actions pl 2: left or right

$u_1(U, L) = a_1$

Player 2

Player 1

	L	R
U	a_1, a_2	b_1, b_2
D	c_1, c_2	d_1, d_2

Prisoner's Dilemma

- The game was first posed by Flood and Dresher at RAND in 1950.
- The game consists of the following elements.
  - **Players**: There are two suspects.
  - **Actions**: Stay quiet or squeal.
  - **Preferences**:
    - Both squeal → they each get 10 years in prison.
    - Both stay quiet → they each get 2 years in prison.
    - One squeals, the other stays quiet → the one that squeals gets 0 years, the other gets 15 years.

Years in prison

P 2

P 1	SQ	S
SQ	10, 10	15, 0
S	0, 15	2, 2

$(S, SQ) >_1 (S, S) >_1 (SQ, SQ) >_1 (SQ, S).$

~~$(S, SQ) \succ_i (SQ, SQ) \succ_i (S, S) \succ_i (SQ, S)$~~

$(SQ, S) >_2 (S, S) >_2 (SQ, SQ) >_2 (S, SQ)$

PRISONER'S DILEMMA (CONT.)

$u_1(S, SQ) > u_1(SQ, SQ)$

$= 3$ $\quad$ $= 2$

0 year $\quad$ 10 years

payoffs.

		Prisoner 2	
		Stay Quiet	Squeal
Prisoner 1	Stay Quiet	2,2	0,3
	Squeal	3,0	1,1

→ these are "positive payoffs": desirable.

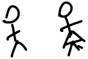
Prisoner's Dilemma (Examples)

		Firm 2	
		High Price	Low Price
Firm 1	High Price	300,300	0,400
	Low Price	400,0	200,200

		Athlete 2	
		Clean	Steroids
Athlete 1	Clean	5,5	2,8-c
	Steroids	8-c,2	5-c,5-c

Battle of the Sexes

- The game was first posed by Luce and Raiffa in 1957. No smartphone → no way to communicate.
- The game consists of the following elements.
    - **Players**: There is a man and a woman.
    - **Actions**: Go to boxing or opera.
    - **Preferences**:
        - Meet at the boxing game → man earns a payoff of 2 and woman of 1.
        - Meet at the opera → woman earns a payoff of 2 and man of 1.
        - Don't meet each other → they each get a payoff of 0.



at work.

Go out tonight

$$(B,B) \succ_1 (O,O) \succ_1 (O,B) \sim_1 (B,O)$$

$$(O,O) \succ_2 (B,B) \succ_2 (O,B) \sim_2 (B,O)$$

Battle of the Sexes (Cont.)

woman

Player 2

		Boxing	Opera
man Player 1	Boxing	$\underline{2},\underline{1}$	0,0
	Opera	0,0	$\underline{1},\underline{2}$

BATTLE OF THE SEXES (EXAMPLES)

		Firm 2	
		Windows	OSX
Firm 1	Windows	20,20-c	10,10
	OSX	10,10	20-c,20

		Firm 2	
		LA	NY
Firm 1	LA	10-c,10	2,2
	NY	2,2	10,10-c

Chicken Game

- The game was first posed by biologist John Maynard Smith in 1973.
- The game consists of the following elements.
    - **Players**: There are two drivers.
    - **Actions**: Go straight or swerve.
    - **Preferences**:
        - If one goes straight and the other swerves → the one that swerved is the chicken.
        - If both swerve → at least they do not crash.
        - If both go straight → they crash.

$$(S, Sw) \succ_i (Sw, Sw) \succ_i (Sw, S) \succ_i (S, S)$$

Chicken (Cont.)

Player 2

Player 1		Swerve	Straight
	Swerve	3,3	2,4
	Straight	4,2	1,1

Chicken (Examples)

		US	
		Compromise	Escalate
USSR	Compromise	3,3	1,5-c
	Escalate	5-c,1	1-c,1-c

		Democrats	
		Raise Debt Ceiling	Keep Debt Ceiling
Republicans	Cut Spending	3,3	2,4
	Keep Current	4,2	1,1

Stag Hunt

- The game was first posed by philosopher Jean-Jacques Rousseau in 1775.
- The game consists of the following elements.
  - **Players**: There are two hunters.
  - **Actions**: Stag or Hare.
  - **Preferences**:
    - Hunt stag solo → the individual gets 0 units of food.
    - Hunt hare solo → the individual gets 1 unit of food.
    - Hunt stag with other player → each gets 2 units of food.

$$(S, S) \succ_i (H, H) \sim_i (H, S) \succ_i (S, H)$$

STAG HUNT (CONT.)

		Player 2	
		Stag	Hare
Player 1	Stag	2,2	0,1
	Hare	1,0	1,1

Stag Hunt (Examples)

		Worker 2	
		High Effort	Low Effort
Worker 1	High Effort	4-c,4-c	1-c,1
	Low Effort	1,1-c	1,1

		Depositor 2	
		Deposit	Withdraw
Depositor 1	Deposit	3,3	0,1
	Withdraw	1,0	1,1

MATCHING PENNIES

- The game was first posed by von Neumann (1928).
- The game consists of the following elements.
  - **Players**: There are two individuals.
  - **Actions**: Choose heads or tails.
  - **Preferences**:
    - Player 1 wins → the actions match.
    - Player 2 wins → the actions do not match.

$$(H,H) \sim_1 (T,T) \succ_1 (H,T) \sim_1 (T,H)$$

$$(H,T) \sim_2 (T,H) \succ_2 (H,H) \sim_2 (T,T)$$

MATCHING PENNIES (CONT.)

Player 2

		Heads	Tails
Player 1	Heads	1,−1	−1,1
	Tails	−1,1	1,−1

MATCHING PENNIES (EXAMPLES)

		Goalie	
		East	West
Kicker	East	-1,1	1, −1
	West	1, −1	-1,1

		Driver	
		Speed	Obey
Policeman	Check	1,-1	-1,1
	Sleep	-1,1	1,-1

NASH EQUILIBRIUM

2 players: 1, 2

Actions 1 : a_1 example {up, down}

2 : a_2 example {left, right}

- An equilibrium is a state in which opposing forces or influences are balanced.

(a_1, a_2) is a profile

(up, right) is a profile.

n players

vector, a profile

- If $\boxed{a}$ is an action profile, $a = (a_1, a_2, \ldots, a_n)$, then a_{-i} is an action profile containing everyone's action except player i, i.e., $a_{-i} = (a_1, a_2, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$.

3 players, $a = (a_1, a_2, a_3)$

$a_{-2} = (a_1, a_3)$ $a_{-1} = (a_2, a_3)$ $a_{-3} = (a_1, a_2)$

Definition

The action profile a^* in a strategic game with ordinal preferences is a **Nash equilibrium** (NE) if for every player i,

$$u_i(a^*) \geq u_i(a_i, a^*_{-i}) \text{ for every action profile } a_i \text{ of player } i,$$

Ex: $a^* = (\text{up}, \text{left}) = (a_1^*, a_2^*)$.

- $u_1(\text{up}, \text{left}) \geq u_1(\text{down}, \text{left})$
- $u_2(\text{up}, \text{left}) \geq u_2(\text{up}, \text{right})$.

where u_i is a payoff function that represents player i's preferences.

→ there is no unilateral profitable deviation.

$\boxed{a^*}$ is an action profile

with two players 1, 2 $a^* = (a_1^*, a_2^*)$

A Nash equilibrium is an action profile a^* such that for every player $i = 1, \dots, n$

$$u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*) \quad ; \quad a_i \neq a_i^*$$

With two players $a^* = (a_1^*, a_2^*)$ is a N.E. if

1) $u_1(a_1^*, a_2^*) \geq u_1(a_1, a_2^*)$ with $a_1 \neq a_1^*$

2) $u_2(a_1^*, a_2^*) \geq u_2(a_1^*, a_2)$ with $a_2 \neq a_2^*$.

Strategic game with ordinal preferences

- players : player 1 and player 2
- set of actions A_i with $i = 1, 2$

$$A_1 = \{ T, D \}$$

$$A_2 = \{ L, R \}$$

- set of preferences : $u_1(T, L) > u_1(T, R) \dots$

---

3 players : 1, 2, 3 $\quad a = (a_1, a_2, a_3)$

$$a_{-2} = (a_1, a_3)$$

$$(a_2, a_{-2}) = (a_1, a_2, a_3)$$

BEST RESPONSE = new tool

- The best response for player i given action(s) a_{-i} is written as:

$$B_i\left(a_{-i}\right) = \left\{a_i \text{ in } A_i : u_i\left(a_i, a_{-i}\right) \geq u_i\left(a_i', a_{-i}\right) \text{ for all } a_i' \text{ in } A_i\right\}.$$

BEST RESPONSE

- The best response for player i given action(s) a_{-i} is written as:

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player 2

Player 1

	L	R
U	2,0	3,3
D	2,5	1,4

$B_1(R) = \{U\}.$

$B_1(L) = \{U, D\}$

$B_2(U) = \{R\}$

$B_2(D) = \{L\}$

BEST RESPONSE

- The best response for player i given action(s) a_{-i} is written as:

$$B_i\left(a_{-i}\right)=\left\{a_i \text{ in } A_i : u_i\left(a_i, a_{-i}\right) \geq u_i\left(a_i', a_{-i}\right) \text{ for all } a_i' \text{ in } A_i\right\}.$$

	L	R
U	2,0	3,3
D	2,5	1,4

$$BR_1\left(L\right)=\left\{U, D\right\}$$

Best Response

- The best response for player i given action(s) a_{-i} is written as:

$$B_i\left(a_{-i}\right)=\left\{a_i \text{ in } A_i : u_i\left(a_i, a_{-i}\right) \geq u_i\left(a_i', a_{-i}\right) \text{ for all } a_i' \text{ in } A_i\right\}.$$

	L	R
U	2,0	3,3
D	2,5	1,4

$$BR_1\left(L\right)=\left\{U, D\right\}$$

$$BR_1\left(R\right)=\left\{U\right\}$$

BEST RESPONSE

- The best response for player i given action(s) a_{-i} is written as:

$$B_i\left(a_{-i}\right) = \left\{a_i \text{ in } A_i : u_i\left(a_i, a_{-i}\right) \geq u_i\left(a_i', a_{-i}\right) \text{ for all } a_i' \text{ in } A_i\right\}.$$

	L	R
U	2,0	3,3
D	2,5	1,4

$$BR_1\left(L\right) = \left\{U, D\right\}$$

$$BR_1\left(R\right) = \left\{U\right\}$$

$$BR_2\left(U\right) = \left\{R\right\}$$

BEST RESPONSE

- The best response for player i given action(s) a_{-i} is written as:

$$B_i\left(a_{-i}\right) = \left\{a_i \text{ in } A_i : u_i\left(a_i, a_{-i}\right) \geq u_i\left(a_i', a_{-i}\right) \text{ for all } a_i' \text{ in } A_i\right\}.$$

	L	R
U	2,0	3,3
D	2,5	1,4

$$BR_1\left(L\right) = \left\{U, D\right\}$$

$$BR_1\left(R\right) = \left\{U\right\}$$

$$BR_2\left(U\right) = \left\{R\right\}$$

$$BR_2\left(D\right) = \left\{L\right\}$$

BEST RESPONSE

- The best response for player i given action(s) a_{-i} is written as:

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	L	R
U	$\underline{2}$,0	3,3
D	$\underline{2}$,5	1,4

$$BR_1\left(L\right) = \left\{U, D\right\}$$

$$BR_1\left(R\right) = \left\{U\right\}$$

$$BR_2\left(U\right) = \left\{R\right\}$$

$$BR_2\left(D\right) = \left\{L\right\}$$

BEST RESPONSE

- The best response for player i given action(s) a_{-i} is written as:

$$B_i\left(a_{-i}\right)=\left\{a_i \text{ in } A_i : u_i\left(a_i,a_{-i}\right)\geq u_i\left(a_i',a_{-i}\right) \text{ for all } a_i' \text{ in } A_i\right\}.$$

	L	R
U	$\underline{2}$,0	$\underline{3}$,3
D	$\underline{2}$,5	1,4

$$BR_1\left(L\right)=\left\{U,D\right\}$$

$$BR_1\left(R\right)=\left\{U\right\}$$

$$BR_2\left(U\right)=\left\{R\right\}$$

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	L	R
U	$\underline{2},0$	$\underline{3},\underline{3}$
D	$\underline{2},5$	$1,4$

$$BR_1\left(L\right)=\left\{U,D\right\}$$

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	L	R
U	$\overset{*}{\underline{2}},0$	$\overset{*}{\underline{3}},\overset{*}{\underline{3}}$
D	$\overset{*}{\underline{2}},\overset{*}{\underline{5}}$	$1,4$

$$BR_1\left(L\right) = \left\{U, D\right\}$$

$$BR_1\left(R\right) = \left\{U\right\}$$

$$BR_2\left(U\right) = \left\{R\right\}$$

$$BR_2\left(D\right) = \left\{L\right\}$$

2 N.E: $(\underset{\uparrow}{D}, \underset{\uparrow}{L})$ and (U, R)

ALTERNATIVE DEFINITION OF A NASH EQUILIBRIUM

Proposition

The action profile a^* *is a* **Nash equilibrium** *of a strategic game with ordinal preferences if and only if every player's action is a best response to the other players' actions; that is,*

$$a_i^* \in B_i\left(a_{-i}^*\right) \text{ for every player } i.$$

- An action profile is a Nash equilibrium if every player's action is best responding to each other.

With 2 players $a^* = (a^*_1, a^*_2)$ a N.E iff

$$a_1^* \in B_1(a_2^*)$$

$$a_2^* \in B_2(a_1^*)$$

NASH EQUILIBRIUM (EXAMPLE)

- Consider the following game consisting of the following elements.
  - **Players:** $\{1, 2, 3, 4, 5, 6, 7\}$
  - **Actions:** $\{A, B, C, D\}$
  - **Payoffs:** represented with u_i.

NASH EQUILIBRIUM (EXAMPLE)

- Consider the following game consisting of the following elements.
  - **Players:** $\{1, 2, 3, 4, 5, 6, 7\}$
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  - **Payoffs:** represented with u_i.
- Consider action profile: $\{A, D, C, D, B, B, A\}$.

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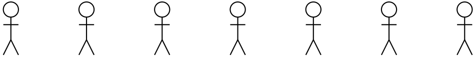

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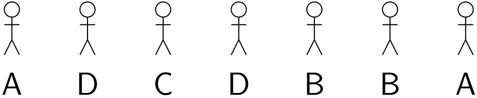

Nash Equilibrium (Example)

- Consider the following game consisting of the following elements.
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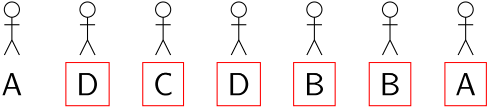

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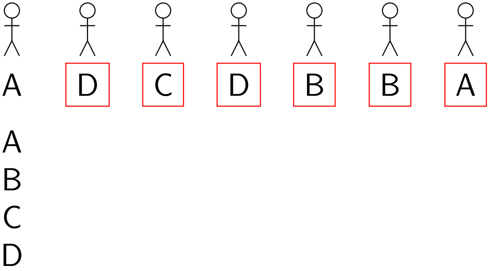

A D C D B B A

A
B
C
D

NASH EQUILIBRIUM (EXAMPLE)

- Consider the following game consisting of the following elements.
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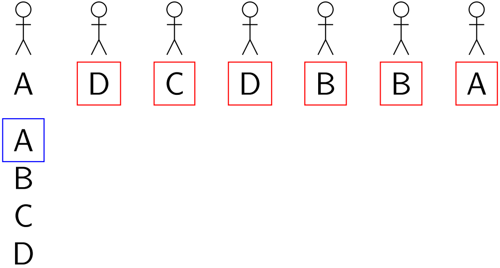
- Consider action profile: $\{A, D, C, D, B, B, A\}$.



A D C D B B A

A
B
C
D

NASH EQUILIBRIUM (EXAMPLE)

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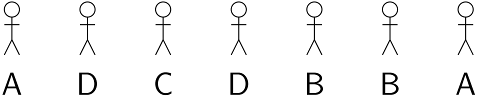

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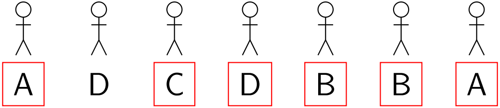

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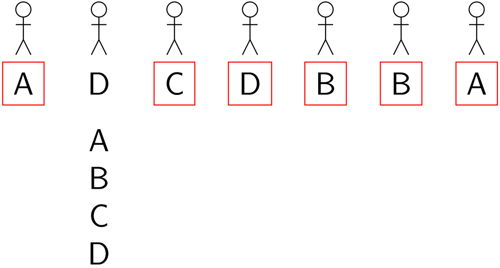
  - **Payoffs:** represented with u_i.
- Consider action profile: $\{A, D, C, D, B, B, A\}$.

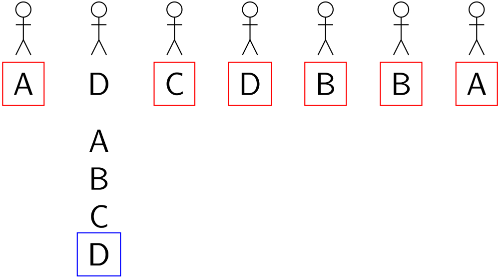


A D C D B B A

A
B
C
D

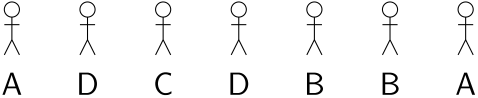
NASH EQUILIBRIUM (EXAMPLE)

- Consider the following game consisting of the following elements.
  - **Players:** $\{1, 2, 3, 4, 5, 6, 7\}$
  - **Actions:** $\{A, B, C, D\}$
  - **Payoffs:** represented with u_i.
- Consider action profile: $\{A, D, C, D, B, B, A\}$.



NASH EQUILIBRIUM (EXAMPLE)

- Consider the following game consisting of the following elements.
  - **Players:** $\{1, 2, 3, 4, 5, 6, 7\}$
  - **Actions:** $\{A, B, C, D\}$
  - **Payoffs:** represented with u_i.
- Consider action profile: $\{A, D, C, D, B, B, A\}$.



NASH EQUILIBRIUM (EXAMPLE)

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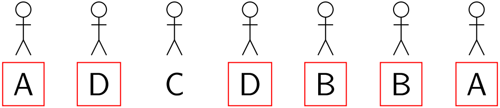
    - **Players:** $\{1, 2, 3, 4, 5, 6, 7\}$
    - **Actions:** $\{A, B, C, D\}$
    - **Payoffs:** represented with u_i.
- Consider action profile: $\{A, D, C, D, B, B, A\}$.



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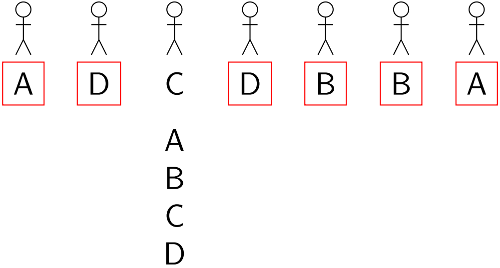
    - **Players:** $\{1, 2, 3, 4, 5, 6, 7\}$
    - **Actions:** $\{A, B, C, D\}$
    - **Payoffs:** represented with u_i.
- Consider action profile: $\{A, D, C, D, B, B, A\}$.



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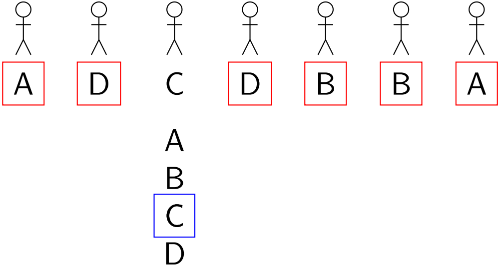

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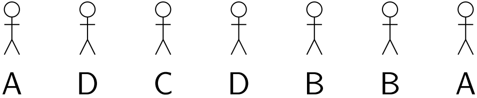

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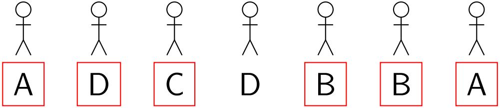

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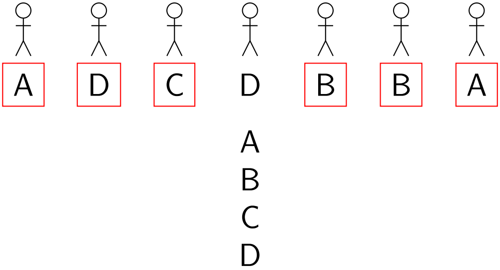
- Consider action profile: $\{A, D, C, D, B, B, A\}$.



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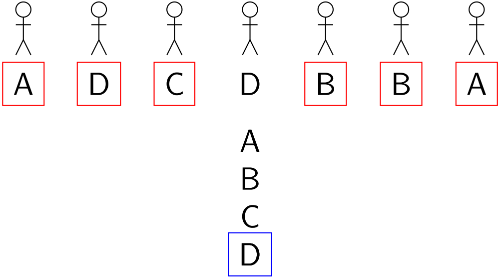

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    - **Payoffs:** represented with u_i.
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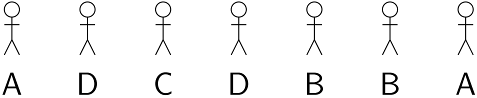

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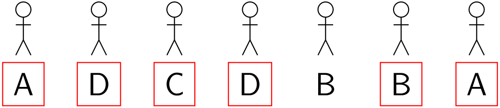
- Consider action profile: $\{A, D, C, D, B, B, A\}$.



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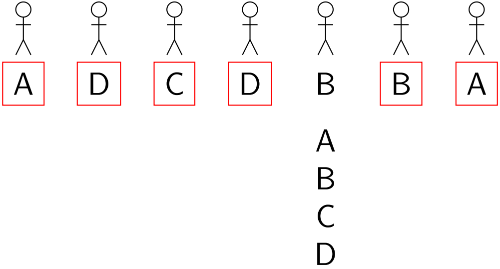

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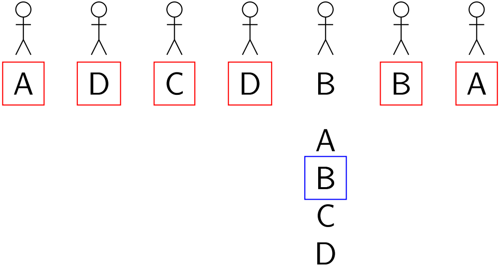

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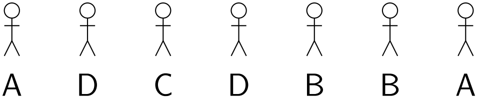
  - **Actions:** $\{A, B, C, D\}$
  - **Payoffs:** represented with u_i.
- Consider action profile: $\{A, D, C, D, B, B, A\}$.



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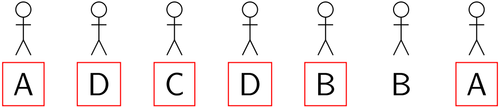

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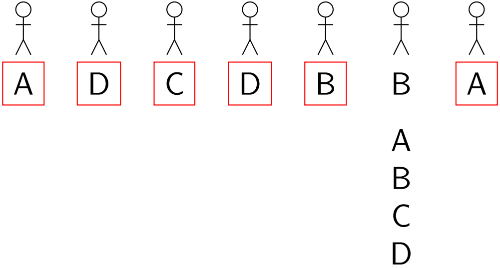
- Consider action profile: $\{A, D, C, D, B, B, A\}$.



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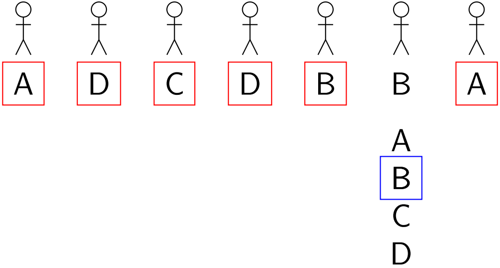

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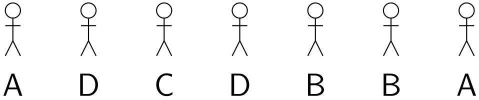

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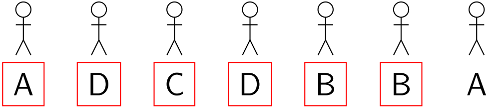
  - **Actions:** $\{A, B, C, D\}$
  - **Payoffs:** represented with u_i.
- Consider action profile: $\{A, D, C, D, B, B, A\}$.



NASH EQUILIBRIUM (EXAMPLE)

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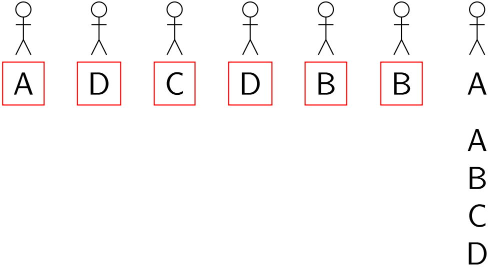

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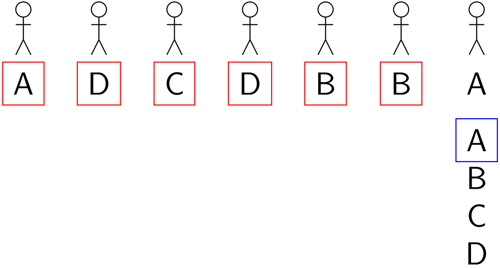

A D C D B B A

A
B
C
D

NASH EQUILIBRIUM (EXAMPLE)

- Consider the following game consisting of the following elements.
  - **Players:** $\{1, 2, 3, 4, 5, 6, 7\}$
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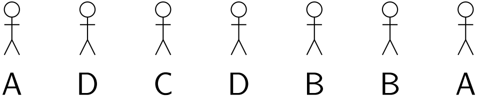

It is in no player's interest to unilaterally deviate from a Nash Equilibrium.

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	* 0,3
	A	* 3,0	** 1,1

N.E: (A,D)

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	** 4,4	1,3
	A	3,1	** 3,3

2NE: (B,C) and (A,D)

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	** 2,5
	A	** 5,2	0,0

2NE: (A,C) and (B,D).

(c) Chicken

		Player 2	
		C	D
Player 1	B	4,3	0,0
	A	0,0	3,4

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0 1,-1	0,4 -1,1
	T	0,4 -1,1	4,0 1,-1

(e) Matching Pennies

How to Find the Nash Equilibrium?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,3
	A	3,0	1,1

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	4,4	1,3
	A	3,1	3,3

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	2,5
	A	5,2	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	4,3	0,0
	A	0,0	3,4

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
	T	0,4	4,0

(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,3
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(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	4,4	1,3
	A	3,1	3,3

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	2,5
	A	5,2	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	4,3	0,0
	A	0,0	3,4

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
	T	0,4	4,0

(e) Matching Pennies

How to Find the Nash Equilibrium?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,1

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	4,4	1,3
	A	3,1	3,3

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	2,5
	A	5,2	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	4,3	0,0
	A	0,0	3,4

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
	T	0,4	4,0

(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

		Player 2	
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Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	4,4	1,3
	A	3,1	3,3

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	2,5
	A	5,2	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	4,3	0,0
	A	0,0	3,4

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
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(e) Matching Pennies

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Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	4,4	1,3
	A	3,1	3,3

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	2,5
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		Player 2	
		C	D
Player 1	B	4,3	0,0
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(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
	T	0,4	4,0

(e) Matching Pennies

How to Find the Nash Equilibrium?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,4	1,3
	A	3,1	3,3

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	2,5
	A	5,2	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	4,3	0,0
	A	0,0	3,4

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
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(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,4	1,3
	A	3,1	$\underline{3}$,3

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	2,5
	A	5,2	0,0

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		Player 2	
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(e) Matching Pennies

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Find all Nash Equilibria in the following games.

		Player 2	
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Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{4}$	1,3
	A	3,1	$\underline{3}$,3

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	2,5
	A	5,2	0,0

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		Player 2	
		C	D
Player 1	B	4,3	0,0
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(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
	T	0,4	4,0

(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,3
	A	3,0	1,1 *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	4,4	1,3
	A	3,1	3,3

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	2,5
	A	5,2	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	4,3	0,0
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(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
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(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{4}$ *	1,3
	A	3,1	$\underline{3}$,$\underline{3}$ *

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	2,5
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Player 1	H	4,0	0,4
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(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

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Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{4}$ *	1,3
	A	3,1	$\underline{3}$,$\underline{3}$ *

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	2,5
	A	$\underline{5}$,2	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	4,3	0,0
	A	0,0	3,4

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
	T	0,4	4,0

(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{4}$ *	1,3
	A	3,1	$\underline{3}$,$\underline{3}$ *

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	$\underline{2}$,5
	A	$\underline{5}$,2	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	4,3	0,0
	A	0,0	3,4

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
	T	0,4	4,0

(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{4}$ *	1,3
	A	3,1	$\underline{3}$,$\underline{3}$ *

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	$\underline{2}$,$\underline{5}$
	A	$\underline{5}$,$\underline{2}$	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	4,3	0,0
	A	0,0	3,4

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
	T	0,4	4,0

(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{4}$ *	1,3
	A	3,1	$\underline{3}$,$\underline{3}$ *

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	$\underline{2}$,$\underline{5}$
	A	$\underline{5}$,$\underline{2}$	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	4,3	0,0
	A	0,0	3,4

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
	T	0,4	4,0

(e) Matching Pennies

How to Find the Nash Equilibrium?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{4}$ *	1,3
	A	3,1	$\underline{3}$,$\underline{3}$ *

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	$\underline{2}$,$\underline{5}$ *
	A	$\underline{5}$,$\underline{2}$ *	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	4,3	0,0
	A	0,0	3,4

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
	T	0,4	4,0

(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

Player 2

Player 1

	C	D
B	2,2	0,$\underline{3}$
A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

Player 2

Player 1

	C	D
B	$\underline{4}$,$\underline{4}$ *	1,3
A	3,1	$\underline{3}$,$\underline{3}$ *

(b) Stag Hunt

Player 2

Player 1

	C	D
B	4,4	$\underline{2}$,$\underline{5}$ *
A	$\underline{5}$,$\underline{2}$ *	0,0

(c) Chicken

Player 2

Player 1

	C	D
B	$\underline{4}$,3	0,0
A	0,0	3,4

(d) Battle of the Sexes

Player 2

Player 1

	H	T
H	4,0	0,4
T	0,4	4,0

(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{4}$ *	1,3
	A	3,1	$\underline{3}$,$\underline{3}$ *

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	$\underline{2}$,$\underline{5}$ *
	A	$\underline{5}$,$\underline{2}$ *	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	$\underline{4}$,3	0,0
	A	0,0	$\underline{3}$,4

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
	T	0,4	4,0

(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{4}$ *	1,3
	A	3,1	$\underline{3}$,$\underline{3}$ *

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	$\underline{2}$,$\underline{5}$ *
	A	$\underline{5}$,$\underline{2}$ *	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{3}$	0,0
	A	0,0	$\underline{3}$,4

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
	T	0,4	4,0

(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{4}$ *	1,3
	A	3,1	$\underline{3}$,$\underline{3}$ *

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	$\underline{2}$,$\underline{5}$ *
	A	$\underline{5}$,$\underline{2}$ *	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{3}$	0,0
	A	0,0	$\underline{3}$,$\underline{4}$

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
	T	0,4	4,0

(e) Matching Pennies

How to Find the Nash Equilibrium?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{4}$ *	1,3
	A	3,1	$\underline{3}$,$\underline{3}$ *

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	$\underline{2}$,$\underline{5}$ *
	A	$\underline{5}$,$\underline{2}$ *	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{3}$ *	0,0
	A	0,0	$\underline{3}$,$\underline{4}$ *

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	4,0	0,4
	T	0,4	4,0

(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{4}$ *	1,3
	A	3,1	$\underline{3}$,$\underline{3}$ *

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	$\underline{2}$,$\underline{5}$ *
	A	$\underline{5}$,$\underline{2}$ *	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{3}$ *	0,0
	A	0,0	$\underline{3}$,$\underline{4}$ *

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	$\underline{4}$,0	0,4
	T	0,4	4,0

(e) Matching Pennies

How to Find the Nash Equilibrium?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{4}$ *	1,3
	A	3,1	$\underline{3}$,$\underline{3}$ *

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	$\underline{2}$,$\underline{5}$ *
	A	$\underline{5}$,$\underline{2}$ *	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{3}$ *	0,0
	A	0,0	$\underline{3}$,$\underline{4}$ *

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	$\underline{4}$,0	0,4
	T	0,4	$\underline{4}$,0

(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

		Player 2	
		C	D
Player 1	B	2,2	0,$\underline{3}$
	A	$\underline{3}$,0	$\underline{1}$,$\underline{1}$ *

(a) Prisoner's Dilemma

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{4}$ *	1,3
	A	3,1	$\underline{3}$,$\underline{3}$ *

(b) Stag Hunt

		Player 2	
		C	D
Player 1	B	4,4	$\underline{2}$,$\underline{5}$ *
	A	$\underline{5}$,$\underline{2}$ *	0,0

(c) Chicken

		Player 2	
		C	D
Player 1	B	$\underline{4}$,$\underline{3}$ *	0,0
	A	0,0	$\underline{3}$,$\underline{4}$ *

(d) Battle of the Sexes

		Player 2	
		H	T
Player 1	H	$\underline{4}$,0	0,$\underline{4}$
	T	0,4	$\underline{4}$,0

(e) Matching Pennies

HOW TO FIND THE NASH EQUILIBRIUM?

Find all Nash Equilibria in the following games.

Player 2

Player 1	C	D
B	[illegible]	0,3
A	3,0	1,1 *

(a) Prisoner's Dilemma

Player 2

Player 1	C	D
B	4,4 *	1,3
A	3,1	3,3 *

(b) Stag Hunt

Player 2

Player 1	C	D
B	4,4	2,5 *
A	5,2 *	0,0

(c) Chicken

Player 2

Player 1	C	D
B	4,3 *	0,0
A	0,0	3,4 *

(d) Battle of the Sexes

Player 2

Player 1	H	T
H	4,0	0,4
T	0,4	4,0

(e) Matching Pennies

Strictly Dominated Strategy

Definition

In a SGWOP, player i's action a_i'', strictly dominates her actions a_i', if

$u_i(a_i'', a_{-i}) > u_i(a_i', a_{-i})$ for every list a_{-i} of the other players' actions,

where u_i is player i's payoff function. We say that the action a_i' is **strictly dominated**.

pl 2

pl 1

	L	R
U	3, 3	1, 1
D	4, 1	2, 2

action U is strictly dominated by D for player 1.

Strictly Dominated Strategy

Definition

In a SGWOP, player i's action a_i'', strictly dominates her actions a_i', if

$u_i\left(a_i'', a_{-i}\right) > u_i\left(a_i', a_{-i}\right)$ for every list a_{-i} of the other players' actions,

where u_i is player i's payoff function. We say that the action a_i' is **strictly dominated**.

	L	R
U	3, 3	1, 1
D	$\underline{4,}$ 1	2, 2

Strictly Dominated Strategy

Definition

In a SGWOP, player i's action a_i'', strictly dominates her actions a_i', if

$u_i\left(a_i'', a_{-i}\right) > u_i\left(a_i', a_{-i}\right)$ for every list a_{-i} of the other players' actions,

where u_i is player i's payoff function. We say that the action a_i' is **strictly dominated**.

	L	R
U	$3,\ 3$	$1,\ 1$
D	$\underline{4,}\ 1$	$\underline{2,}\ 2$

Strictly Dominated Strategy

Definition

In a SGWOP, player i's action a_i'', strictly dominates her actions a_i', if

$u_i\left(a_i'', a_{-i}\right) > u_i\left(a_i', a_{-i}\right)$ for every list a_{-i} of the other players' actions,

where u_i is player i's payoff function. We say that the action a_i' is **strictly dominated**.

	L	R
U	3, $\underline{3}$	1, 1
D	$\underline{4,}$ 1	$\underline{2,}$ 2

Strictly Dominated Strategy

Definition

In a SGWOP, player i's action a_i'', strictly dominates her actions a_i', if

$$u_i\left(a_i'', a_{-i}\right) > u_i\left(a_i', a_{-i}\right)\text{ for every list } a_{-i} \text{ of the other players' actions,}$$

where u_i is player i's payoff function. We say that the action a_i' is **strictly dominated**.

	L	R
U	$3, \underline{3}$	$1, 1$
D	$\underline{4}, 1$	$\underline{2}, \underline{2}$

Strictly Dominated Strategy

Definition

In a SGWOP, player i's action a_i'', strictly dominates her actions a_i', if

$u_i\left(a_i'', a_{-i}\right) > u_i\left(a_i', a_{-i}\right)$ for every list a_{-i} of the other players' actions,

where u_i is player i's payoff function. We say that the action a_i' is **strictly dominated**.

	L	R
U	$3, \underline{3}$	$1, 1$
D	$\underline{4,}\ 1$	$\underline{2,}\ \underline{2}$

- U is strictly dominated by D.

Strictly Dominated Strategy

Definition

In a SGWOP, player i's action a_i'', strictly dominates her actions a_i', if

$u_i\left(a_i'', a_{-i}\right) > u_i\left(a_i', a_{-i}\right)$ for every list a_{-i} of the other players' actions,

where u_i is player i's payoff function. We say that the action a_i' is **strictly dominated**.

	L	R
U	$3,\ \underline{3}$	$1,\ 1$
D	$\underline{4,}\ 1$	$\underline{2,}\ \underline{2}$

- U is strictly dominated by D.
- Neither L nor R are strictly dominated.

Strictly Dominated Strategy

Definition

In a SGWOP, player i's action a_i'', strictly dominates her actions a_i', if

$u_i(a_i'', a_{-i}) > u_i(a_i', a_{-i})$ for every list a_{-i} of the other players' actions,

where u_i is player i's payoff function. We say that the action a_i' is **strictly dominated**.

	L	R
U	3, $\underline{3}$	1, 1
D	$\underline{4}$, 1	$\underline{2}$, $\underline{2}$

- U is strictly dominated by D.
- Neither L nor R are strictly dominated.
- A strictly dominated ~~strategy~~ action will never be played in a Nash equilibrium.

WEAKLY DOMINATED STRATEGY

Definition

In a SGWOP, player i's action a_i'', weakly dominates her actions a_i', if

$u_i\left(\underline{a_i''}, a_{-i}\right) \geq u_i\left(\underline{a_i'}, a_{-i}\right)$ for every list a_{-i} of the other players' actions,

and,

$u_i\left(a_i'', a_{-i}\right) > u_i\left(a_i', a_{-i}\right)$ for at least one list a_{-i} of the other players'actions,

where u_i is player i's payoff function. We say that the action a_i' is **weakly dominated**.

! in a N.E. a weakly dominated action can be played.

EXAMPLE

2.

1.

	A	~~B~~	C
Z	✱ 3, ✱ 4	~~6, 3~~	✱ 5, 2
Y	✱ 3, 2	~~5, 1~~	2, 3 ✱
~~X~~	~~2, 3~~	~~2, 2~~	~~2, 1~~

Find all:

(i) weakly dominated strategies,

✓

(ii) strictly dominated strategies,

✓

(iii) Nash Equilibria.

✓

		Pl 2	
		B	R
Pl 1	B	7, 3	7, 4
	R	3, 4	10*, 5*

$B_1(B) = a_1$

$B_1(R) = a_1'$

70%

Example 39.1. a synergistic relationship.

players : 1, 2

actions : $a_i \in \mathbb{R}^+$. The effort devoted to the join project.

preferences : $u_i(a_i, a_j) = \underbrace{a_i(c + a_j - a_i)}_{a_1(c+a_2-a_1)}$ with $c>0$ a constant.

Best response : $B_1(a_2)$ → we need to maximize $u_1(a_1, \bar{a}_2)$

$$\max_{a_1} u_1(a_1).$$

$$u_1 = a_1 c + a_1 a_2 - a_1^2 \qquad u_2 = a_2 c + a_1 a_2 - c_2^2$$

$$u_1' = c + a_2 - 2a_1 = 0 \iff a_1 = \frac{c+a_2}{2}$$

$$B_1(a_2) = \frac{c+a_2}{2} = a_1^*$$

Now $B_2(a_1)$.

$$u_2 = a_2(c + a_1 - a_2)$$

$$= a_2 c + a_1 a_2 - a_2^2$$

$$u_2' = \frac{du_2}{da_2} = c + a_1 - 2a_2 = 0$$

$$\Leftrightarrow B_2(a_1) = \frac{c+a_1}{2} = a_2^*$$

$$a_1^* = B_1(a_2^*)$$

$$a_2^* = B_2(a_1^*).$$

$$\begin{cases} a_1^* = \frac{c}{2} + \frac{a_2^*}{2} \\ a_2^* = \frac{c}{2} + \frac{a_1^*}{2} \end{cases}$$

$$\begin{cases} a_1 - \frac{a_2}{2} = \frac{c}{2} & (1) \\ -\frac{a_1}{2} + a_2 = \frac{c}{2} & (2) \end{cases} \Leftrightarrow a_1^* = c = a_2^*.$$

STRICT NASH EQUILIBRIUM

Definition

The action profile a^* in a SGWOP is a **strict Nash equilibrium**, if for every player i,

$$u_i\left(a^*\right) > u_i\left(a_i, a^*_{-i}\right) \text{ for every action profile } a_i \text{ of player } i,$$

where u_i is a payoff function that represents player i's preferences.

STRICT NASH EQUILIBRIUM (EXAMPLE)

	L	R
U	0, 0	2, 1
D	3, 2	0, 2

- The game has 2 Nash equilibria.
- Only 1 Nash equilibrium is strict.
- A Nash equilibrium might consist of weakly dominated strategies.
- The non-strict Nash equilibrium is less stable.

STRICT NASH EQUILIBRIUM (EXAMPLE)

	L	R
U	0, 0	2, 1
D	$\underline{3,}$ 2	0, 2

- The game has 2 Nash equilibria.
- Only 1 Nash equilibrium is strict.
- A Nash equilibrium might consist of weakly dominated strategies.
- The non-strict Nash equilibrium is less stable.

STRICT NASH EQUILIBRIUM (EXAMPLE)

	L	R
U	$0,\ 0$	$\underline{2,}\ 1$
D	$\underline{3,}\ 2$	$0,\ 2$

- The game has 2 Nash equilibria.
- Only 1 Nash equilibrium is strict.
- A Nash equilibrium might consist of weakly dominated strategies.
- The non-strict Nash equilibrium is less stable.

STRICT NASH EQUILIBRIUM (EXAMPLE)

	L	R
U	0, 0	$\underline{2,}$ $\underline{1}$
D	$\underline{3,}$ 2	0, 2

- The game has 2 Nash equilibria.
- Only 1 Nash equilibrium is strict.
- A Nash equilibrium might consist of weakly dominated strategies.
- The non-strict Nash equilibrium is less stable.

STRICT NASH EQUILIBRIUM (EXAMPLE)

	L	R
U	0, 0	$\underline{2,}$ $\underline{1}$
D	$\underline{3,}$ $\underline{2}$	0, $\underline{2}$

- The game has 2 Nash equilibria.
- Only 1 Nash equilibrium is strict.
- A Nash equilibrium might consist of weakly dominated strategies.
- The non-strict Nash equilibrium is less stable.

SYMMETRIC GAMES

Definition

A two-player SGWOP is **symmetric** if the players' set of actions are the same and the players' preferences are represented by payoff function u_1 and u_2 for which $u_1(a_1, a_2) = u_2(a_2, a_1)$ for every action pair (a_1, a_2).

- Players are all homogeneous and no roles are assigned.

Definition

An action profile a^* in a symmetric SGWOP is a **symmetric Nash equilibrium** if it is a Nash equilibrium and a_i^* is the same for every player i.

EXAMPLE

	A	B	C
Z	1, 1	2, 1	4, 1
Y	1, 2	5, 5	3, 6
X	1, 4	6, 3	0, 0

Find all:

(i) Nash Equilibria,

(ii) symmetric Nash Equilibria.