

Introduction to Game Theory

Mid Term

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1. (Total 5 points) (**Dominated Strategies**) Consider the normal form game described by the matrix below :

		Player 2			
		a	b	c	d
Player 1	A	(1,1)	(1,1)	(1,4)	(2,2)
	B	(3,0)	(5,2)	(2,3)	(4,2)
	C	(1,1)	(2,2)	(1,4)	(8,2)
	D	(0,9)	(2,4)	(1,7)	(3,8)

- (a) (3 points) Show that a unique action profile survives by iteratively eliminating strictly dominated actions.
- (b) (2 points) Determine the unique Nash equilibrium (in pure strategies) of this game and show that this equilibrium is not Pareto-optimal.
2. (Total 10 points) (**Bertrand Competition**) Suppose that we have two (duopoly) firms that set prices in a market whose demand curve is given by

$$Q = 4 - p$$

where p is the lower of the two prices ($p = \min\{p_1, p_2\}$). If there is a lower-priced firm, then it meets all of the demand. For example, if $p_1 = 1$ and $p_2 = 2$, then firm 1 meets the entire demand $Q = 4 - 1 = 3$. If the two firms post the same price p , they each get half the market, that is, they each get $\frac{4-p}{2}$. Suppose that prices can only be quoted in dollar units (0, 1, 2, 3, or 4 dollars) and that costs of production are zero. The profit is simply given by the price times the quantity sold by the firm. For example, if $p_1 = 1$ and $p_2 = 2$, $\pi_1 = 1 \times 3 = 3$ and $\pi_2 = 0$.

- (a) (2 points) Write down the normal form of this game, where firm 1 is the row player and firm 2 the column player. Payoffs correspond to the profit made by the firms.
- (b) (2 points) Show that posting a price of 0 dollar and posting a price of 4 dollars are both dominated strategies.
- (c) (2 points) Is there a price that is not a dominated strategy?
- (d) (1 point) Show that the best response to the rival posting a price of 4 dollars is to post a price of 2 dollars.
- (e) (1 point) What is the best response to a price set at 2 dollars by the rival?
- (f) (2 points) What is the Nash equilibrium of this game?
3. (Total 5 points) **You must choose one and only one of the two following questions (a) or (b) :**

- (a) Show graphically that the game described by the matrix below has an infinite amount of mixed-strategy Nash equilibria.¹

		Player 2	
		C	D
Player 1	X	(10, 10)	(2, 7)
	Y	(10, 0)	(3, 1)

- (b) Suppose two firms compete à la Cournot facing an inverse demand function $p(Q) = d - b(Q)$ with $Q = q_1 + q_2$ with $q_1 + q_2 < d/b$ and q_i the quantity produced by firm $i = 1, 2$. Each firm has the same total cost function $C(q_i) = \frac{q_i^2}{2}$, i.e., half the square of the quantity produced. Determine both Best Reply functions and the corresponding Nash Equilibrium, assuming the hypotheses for an interior solution are met.

1. Remember that between any two real numbers x and y with $x < y$ there is an infinite amount of real numbers.