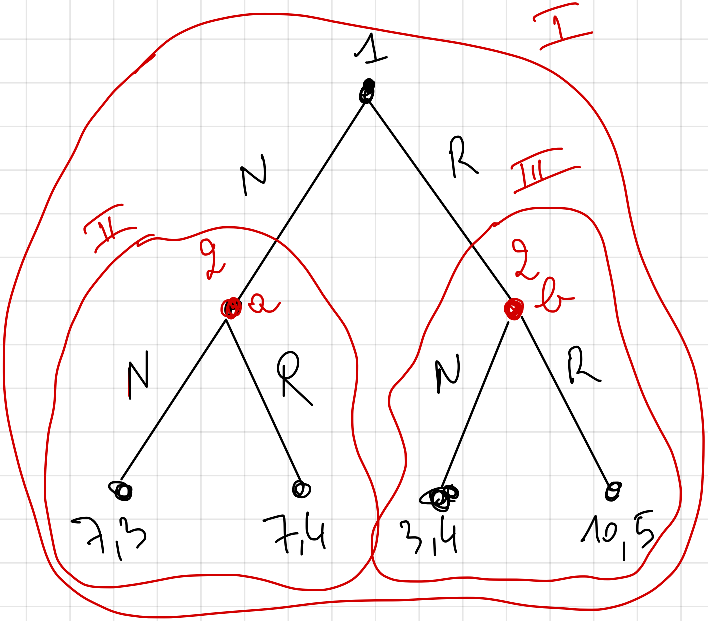
course #9
March 26th.
2026.



two players: 1 and 2

Set of strategies $S_1 = \{N, R\}$

$S_2 = \{NN, RR, NR, RN\}$

player 2

	NN	RR	NR	RN
N	$7^*,3$	$7,4^*$	$7,3$	$7^*,4^*$
R	$3,4$	$10^*,5^*$	$10^*,5^*$	$3,4$

player 1

A strategy for player 2 consists in choosing an action for each node where she has to play.

a	b
N	R

Subgame perfect Nash equilibrium $\rightarrow$ it has to be a N.E in each subgame, i.e. including the whole game (I)

SPNE: (R, RR). → payoffs (10, 5).

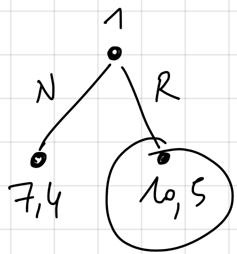
Backward induction.

NE: (R, NR) SP? No. because N is not optimal at node a (subgame II) for player 2.

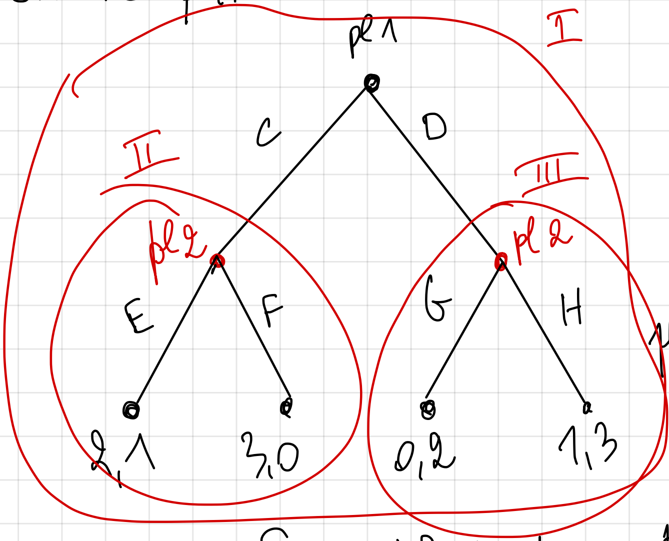
Subgame II : pl 2 → R → 7,4

III : pl 2 → R → 10,5

Now, work with the following reduced game:



osborne p170.



$S_1 = \{C, D\}$

$S_2 = \{EG, EH, FG, FH\}$

player 2

player 1

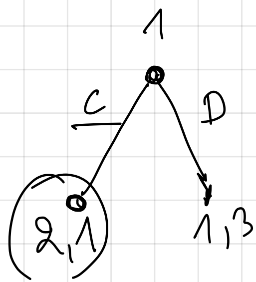
	EG	EH	FG	FH
C	$2^*, 1^*$	$2^*, 1^*$	$3^*, 0$	$3^*, 0$
D	0, 2	$1, 3^*$	0, 2	$1, 3^*$

Two N.E in pure strategies: (C, EG) and (C, EH).

SPNE : backward induction

II : pl 2 $\longrightarrow$ E $\longrightarrow$ 2,1

III : pl 2 $\longrightarrow$ H $\longrightarrow$ 1,3



CHAPTER 6 : Extensive games with perfect information
Illustrations

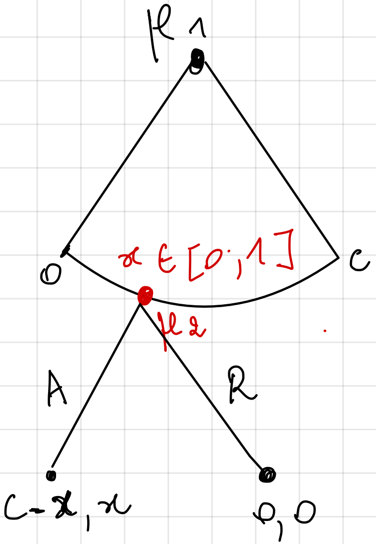
The ultimatum game

- 2 players : 1 and 2
- Bargaining over the division of a "pie" let's say c € (100 €).

1) player 1 proposes a split x for player 2, $c-x$ player 1.
2) player 2 accepts or rejects.

- final payoffs : $(c-x, x)$ or $(0,0)$.
- Terminal histories (x, Z) where

- $x \in [0 ; c]$ $\quad c \in \mathbb{R}^+$.
- $Z \in \{\text{Accept}, \text{reject}\}$

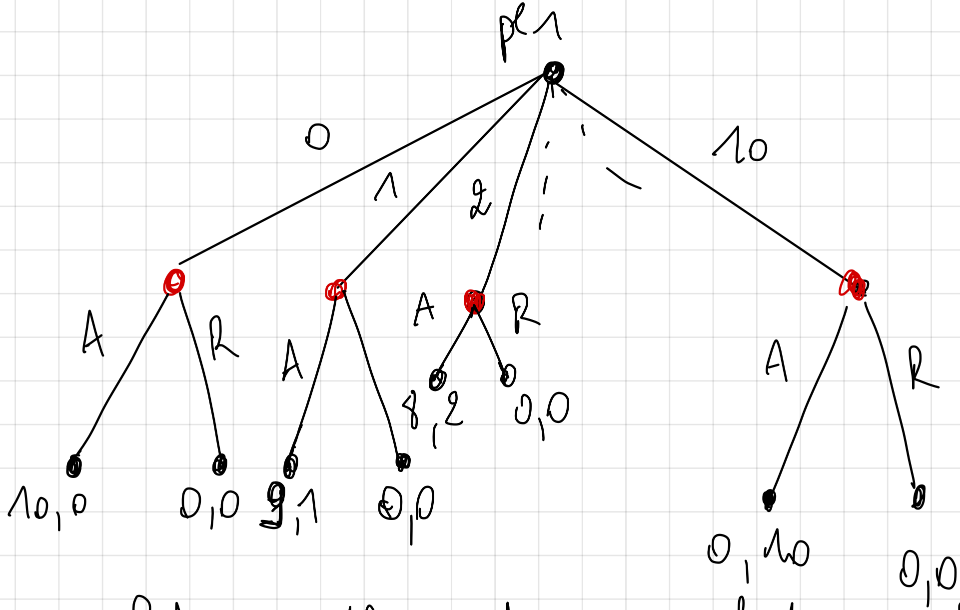


- Player function $P(\phi) = Pl\,1$
  $P(x) = Pl\,2$

- $u_1(x, \text{Accept}) = c - x$
- $u_2(x, \text{Accept}) = x$

- $u_i(x, \text{Reject}) = 0$
  $i = 1, 2$

Discrete version :



$c = 10$

What are the subgame perfect NE ?

Backward induction , look at subgames of length 1 :

at each such subgame , player 2 faces $x \in [0;10]$.

- If $x > 0$: she accepts.
- If $x = 0$: she is indifferent between R or A.

In a SPNE, player 2 accepts everything (including 0)

or accepts $x > 0$ and rejects $x = 0$.

whole game, optimal strategy for player 1.

- If player 2 accepts everything, then it's optimal for Pl. 1 to propose $x = 0$. $\rightarrow$ final payoff $10, 0$.
- If player 2 accepts $x > 0$ and reject $x = 0$, then Pl 1 proposes $x = 1$. $\rightarrow$ final payoff $9, 1$.

Back to the continuous game $C \in \mathbb{R}^+$, and $x \in [0; C]$.

For player 2, it does not change anything:

Player 1:
- if player 2 accepts everything, Player 1 proposes $x = 0$
- if player 2 accepts $x > 0$ and reject $x = 0$, there does

not exist an optimal strategy for player 1.

The only weak SPNE : $x = 0$, player 2 accepts everything.

Experiments : 70's.

- $x \simeq$ 65 or 70 % of the amount.
- about 20% of the offers were rejected.

→ Equity. 50/50.

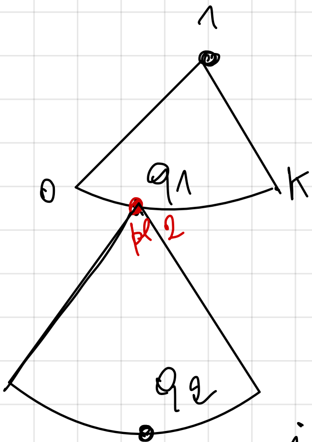
→ risk aversion / uncertainty.

Milton Friedman : good model, hypothesis may be irrealistic as long as the prediction is good.

<u>Stackelberg Model</u> : sequential Cournot.

2 firms, 1 is the leader, and 2 is the follower.

firm$_1$ set its quantity first : q_1. Then firm 2 sets q_2.



. Terminal histories (q_1, q_2).

. payoffs : $Q = q_1 + q_2$

$P(Q)$ = inverse demand

$c_i(q_i)$: individual total cost

$i = 1,2$: $\pi_i(q_i, q_{-i}) = q_i P(Q) - C_i(q_i)$.

Simplify calculations: $P(Q) = 2 - Q$
$= 2 - q_1 - q_2$

$C_i(q_i) = q_i$ (Constant marginal cost $= 1$).

SPNE, backward induction

quantity × price − total cost.

Firm 2: $\max \pi_2 = q_2(2 - q_1 - q_2) - q_2$

$= 2q_2 - q_1 q_2 - q_2^2 - q_2$

$= q_2 - q_1 q_2 - q_2^2$

$$\frac{\partial \pi_2}{\partial q_2} = 1 - q_1 - 2q_2 = 0 \iff \boxed{q_2^* = \frac{1 - q_1}{2}}$$

Now, we look at the first stage, player 1 has to choose q_1 knowing that $q_2 = \frac{1 - q_1}{2}$.

Firm 1's problem:

$$\max \pi_1 = q_1(2 - q_1 - q_2) - q_1$$

$$= q_1\left(2 - q_1 - \left(\frac{1-q_1}{2}\right)\right) - q_1$$

$$= 2q_1 - q_1^2 - q_1/2 + q_1^2/2 - q_1$$

$$= \frac{q_1}{2} - \frac{q_1^2}{2}$$

$$\frac{\partial \pi_1}{\partial q_1} = \frac{1}{2} - \frac{2q_1}{2} = 0 \Leftrightarrow q_1^* = \frac{1}{2}$$

So, $q_2^* = \frac{1-q_1}{2}$

$$= \frac{1-1/2}{2} = \frac{1}{4}.$$

SPNE $\left(\frac{1}{2}; \frac{1}{4}\right)$.

$\rightarrow \pi_1 > \pi_2$.

Next week: ch 9 Bayesian games.