Course #10.

April, 2. 2026.

CHAP 9 BAYESIAN GAMES

More realistic

Uncertainty or incomplete information about your "competitor"

Motivation

2 players: 1 and 2

player 2 may be of type A or type B

player 1 has only one type, and she does not know of what type player 2 is.

pl 2 type A

pl 1	L	R
L	3 , 1	2 , 0
R	0 , 1	4 , 0

pl 2 type B

pl 1	L	R
L	3 , 0	2 , 1
R	0 , 0	4 , 1

1. Suppose you are pl 2 type A
   L or R? L
2. Suppose you are pl 2 type B
   L or R? R
3. If you are pl 1: L or R

pl 1 L : 3 or 2 [2 ; 3]

pl 1 R : 0 or 4 [0 ; 4]

It depends on the realization of the type of Player 2.

Example 273.1 Variant of BoS.

2 players ; player 2 → meet player 1 / → avoid player 2

(1)

Proba 1/2

	pl 2: B	S
pl 1: B	2 , 1	0 , 0
S	0 , 0	1 , 2

player 2 wishes to meet pl 1
"type Y"

Proba 1/2

	pl 2: B	S
pl 1: B	2 , 0	0 , 2
S	0 , 1	1 , 0

player 2 wishes to avoid pl 1
"type N"

- preferences are over lotteries : Bernoulli payoffs
- Two states (of world) , each state coincide with a given type for player 2.

when they play : player 2 knows her type , player 1 doesn't.

→ player 1 must form $\boxed{\text{beliefs}}$ about likelihood of the types of player 2.

→ pl 1 will calculate expected payoffs.

For example, if we suppose pl 2 type Y chooses B and pl 2 type N chooses S, then

$$E_1(B) = \frac{1}{2}2 + \frac{1}{2}0 = 1$$

$$E_1(S) = \frac{1}{2}0 + \frac{1}{2}1 = \frac{1}{2}.$$

- suppose pl 2 always chooses B

$$E_1(B) = \frac{1}{2}2 + \frac{1}{2}2 = 2$$

$$E_1(S) = \frac{1}{2}0 + \frac{1}{2}0 = 0.$$

Table of expected payoffs for pl 1 :

	(B,B)	(B,S)	(S,B)	(S,S)
B	2	1	1	0
S	0	1/2	1/2	1

A pure Nash equilibrium must specify three actions :

- one for player 1
- one for $\underbrace{\text{each type}}_{2}$ of player 2

Steady state : - optimal for player 1 given the actions of the two types of player 2 and her beliefs.

- optimal for each type of player 2 given the action of player 1.

In this game, $(B,(B,S))$ is a N.E.

is $(B,(S,B))$ a N.E? No.

We consider another variant of the BoS game:

Both players have two types.

Beliefs: - player 1 thinks that player 2 is of type Y_2 with probability 1/2 and - - - - - - - - - - - . N_2 with probability 1/2.

- player 2 thinks that player 1 is of type Y_1 with probability 2/3 and of type N_1 with probability 1/3.

In this game, there are 4 different possible states:

Y_1Y_2, Y_1N_2, N_1N_2, N_1Y_2.

Player 1 cannot distinguish between states Y_1Y_2 and Y_1N_2

- states N_1Y_2 and N_1N_2

pl 1

	B	1/2	S
B	2,1		0,0
2/3			
S	0,0		1,2

State $Y_1 Y_2$

	B	1/2	S
B	2,0		0,2
2/3			
S	0,1		1,0

State $Y_1 N_2$

pl 1

	B	1/2	S
B	0,1		2,0
1/3			
S	1,0		0,2

State $N_1 Y_2$

	B	1/2	S
B	0,0		2,2
1/3			
S	1,1		0,0

State $N_1 N_2$

pl 2 pl 2

Claim:

((B,B); (B,S)) is a N.E.

and

((S,B); (S,S)).

Exercise 277.1.

$$\left(\overset{Y_1 \, \& N_1}{(B,B)} \; ; \; \boxed{(B,S)} \right)$$

pl 1 type $Y_1 \to B$
pl 1 type $N_1 \to B$

pl 2 type $Y_2 \to B$
pl 2 type $N_2 \to S$

construct a table of expected payoff for $\boxed{N_1}$, Y_2 and N_2.

Y_1 has been done.

Check that $\left(\overset{Y_1 \; N_1}{(B,B)} \; ; \; \overset{Y_2 \; N_2}{(B,S)} \right)$ is N.E.

1) $\boxed{Y_1}$ given that pl 2 (B, S) (Y_2, N_2)   does Y_1 has an incentive to deviate from B ?

2) $\boxed{N_1}$ given that (B, S) does N_1 has an incentive to deviate from B ?

type N_1

	B,B	B,S	S,B	S,S
B	.	–	–	–
S	–	–	.	–

$$\mathbb{E}_{N_1}(B) = \frac{1}{2}0 + \frac{1}{2}2 = 1$$

3) Y_2

4) N_2

9.2. General Definitions

- set of players
- set of states
- set of actions for each player
- set of signals that a player may receive and a signal function that associates a signal with each state.
- For each signal, a belief about states consistent with this signal.
- Bernoulli payoffs (a, ω) with a a profile of actions and ω is a state.

A state ω, player i receives a signal $\tau_i(\omega)$

$$\tau_i : \underset{\text{(a state)}}{\omega} \longrightarrow \underset{\text{(a signal)}}{t}$$

For each ω, $\tau_i(\omega)$ is unique.

$$\tau_1(Y_1 N_2) = \tau_1(Y_1 Y_2).$$