course #11

April 8th 2026.

CH8 Bayesian games

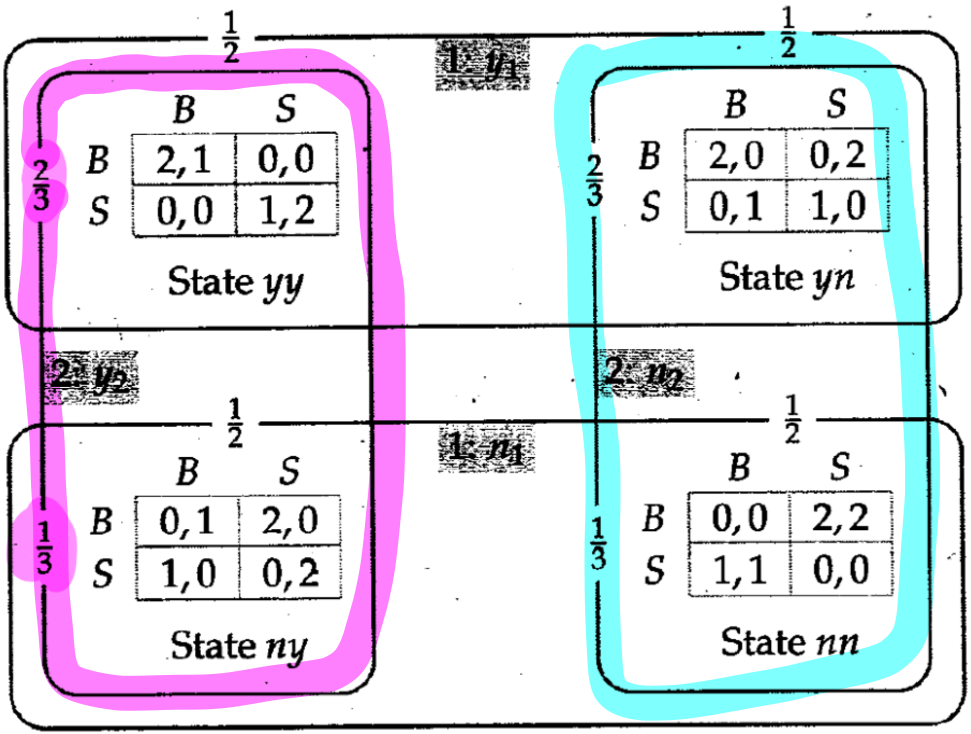
Signals

$\tau_1(yy) = \tau_1(yn) = y_1$

$\tau_1(ny) = \tau_1(nn) = n_1$

$\tau_2(yy) = \tau_2(ny) = y_2$

$\tau_2(yn) = \tau_2(nn) = n_2$



State yy

	B	S
B	2,1	0,0
S	0,0	1,2

State yn

	B	S
B	2,0	0,2
S	0,1	1,0

State ny

	B	S
B	0,1	2,0
S	1,0	0,2

State nn

	B	S
B	0,0	2,2
S	1,1	0,0

Priors on beliefs.

↳ for y_1: two states yy and yn. Player 1 must assign some probabilities to these states. 1/2 and 1/2.

$\tau_2(yy) = y_2$

$\tau_2(ny) = y_2$

n_1 : ny and nn $\to$ 1/2 each

y_2 is $\underbrace{\text{associated to yy and ny}}$. Player 2 assigns

2/3 to yy and 1/3 to ny.

n_2 : yn and nn, player 2 assigns

2/3 for yn and 1/3 to nn.

- In decision theory, one person, more information is better.
- It's not necessarily the case with several players: more information may be detrimental.

$\frac{1}{2}$ 1 $\frac{1}{2}$

$\frac{1}{2}$ 2 $\frac{1}{2}$

	L	M	R
T	1,2ϵ	1,0	1,3ϵ
B	2,2	0,0	0,3

State ω_1

	L	M	R
T	1,2ϵ	1,3ϵ	1,0
B	2,2	0,3	0,0

State ω_2

$0 < \varepsilon < \frac{1}{2}$

$$\tau_1(\omega_1) = \tau_1(\omega_2) = \omega$$

$$\tau_2(\omega_1) = \tau_2(\omega_2) = \omega$$

Unique best response for player 2 is L;

If player 1 chooses T: $\boxed{\text{with L pl 2 gets } \frac{1}{2} 2\epsilon + \frac{1}{2} 2\epsilon = 2\epsilon.}$

with M $\quad \frac{1}{2} 0 + \frac{1}{2} 3\epsilon = \frac{3}{2}\epsilon$

with R $\quad \frac{1}{2} 3\epsilon + \frac{1}{2} 0 = \frac{3}{2}\epsilon$.

If player 1 chooses B: $\boxed{\text{with L pl 2 gets } 2}$

M $\quad 3/2$

R $\quad 3/2$

L is a strictly dominating action for player 2 knowing that, pl 1 chooses B.

The only N.E is (B, L), with payoffs $\boxed{(2, 2)}$.

New game, extra information for player 2: $\mathcal{T}_2(\omega_1) \neq \mathcal{T}_2(\omega_2)$

She can distinguish both states

NE : $(\underline{T}, \underline{(R, M)})$ → payoffs $(1 ; 3\epsilon)$

$0 < \epsilon < \frac{1}{2}$

$3\epsilon < 2$

1,5

$\frac{1}{2}$ $\frac{1}{2}$

1

2

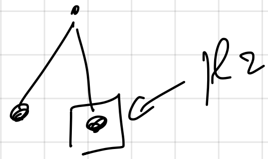
	L	M	R
T	$1, 2\epsilon$	$1, 0$	$1, 3\epsilon$ * *
B	$2, 2$	$0, 0$	$0, 3$ *

State ω_1

	L	M	R
T	$1, 2\epsilon$	$1, 3\epsilon$ * *	$1, 0$
B	$2, 2$	$0, 3$ *	$0, 0$

State ω_2

CH10. Extensive games with imperfect information



- set of players
- set of terminal histories
- player function defined on histories that are not terminal.

$$P(h) = i$$

- preferences → payoffs.

Imperfect information: let H_i the set of histories after which player i moves. H_i will be partitioned.

MATH. A partition of a set of indivisible units is like dividing a cake:

set $= \{a, b, c\}$. The possible partitions are:

1) $\{a\};\{b\};\{c\}$

2) $\{a,b\};\{c\}$

3) $\{a,c\};\{b\}$

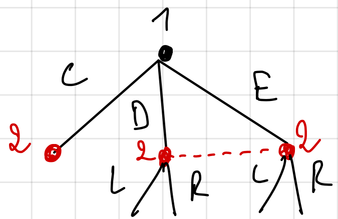
4) $\{b,c\};\{a\}$

5) $\{a,b,c\}$.

To specify i's information, we partition H_i into a collection of information sets $\to$ they form the information partition for player i.

Example.

$H_2 = \{C, D, E\}$.



player 2 has two information sets:

$\{C\}$ and $\{D;E\}$.

Player 2 knows if history C has occured

or if histories D or E has occured

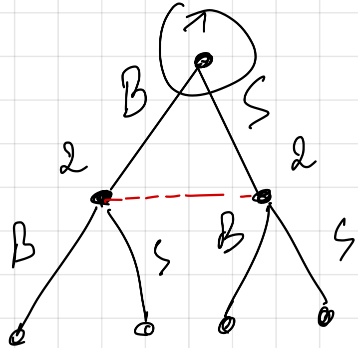
and she cannot discriminate between D or E.

$A(h)$ is the set of possible actions available to the player after history h.

We need $A(h) = A(h')$ if h and h' belong the same information set.

$$A(D) = A(E).$$

Example : BoS



- Histories (B,B) ; (B,S) ; (S,B) ; (S,S)
- $P(\phi) = 1$

  $P(B) = P(S) = 2$
- Information partitions :

  Player 1 : ϕ.

  Player 2 : $\{B ; S\}$.

Example : card game

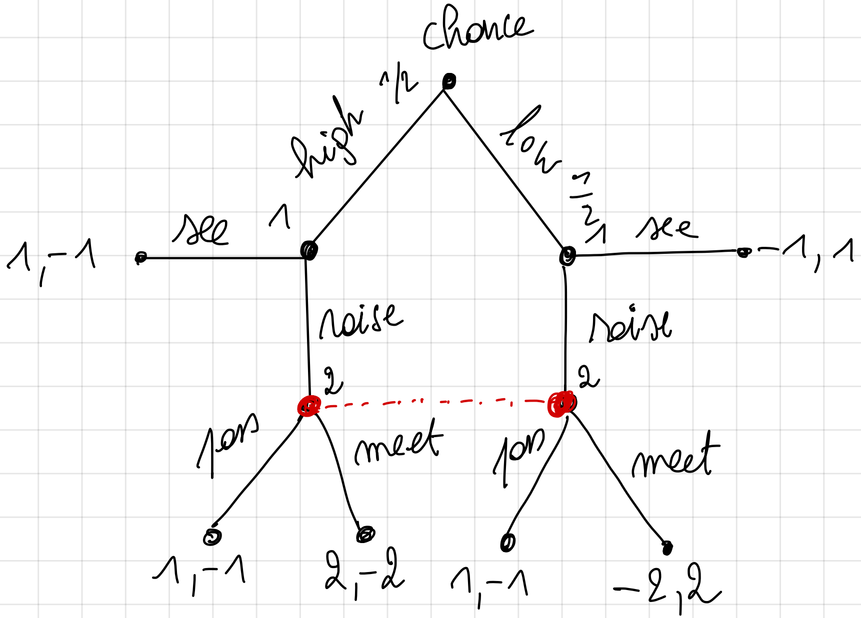
2 players. They both put 1 $ in a pot.

Then player 1 deals a card which can be of high or low value. Player 2 has fixed card (with medium value).

10

0

5



If pl 1 raises, she adds another 1 $ in the pot

Then it's pl 2 move, she may pass or meet (put 1$ extra).

Information partitions:

pl 1 has two information sets: {high} ; {low}.
pl 2 has one information set: {(high, raise) ; (low, raise)}

→ calculate the N.E.