# Introduction to Game Theory

Final Exam

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**Duration : 3 hours. Total points : 100. No documents or electronic devices are allowed, justify all your answers, write legibly and clearly, respect the structure of the exam, number your pages, Any attempt at fraud will result in immediate disqualification.**

## Part I

1. (10 points) Tell and explain the four assumptions needed for a preference relation defined over lotteries over outcomes to be represented by the expected value of a cardinal utility function over deterministic outcomes.
2. (10 points) Tell the definition of an extensive game with imperfect information. Illustrate all the notions used in the definition with an example.

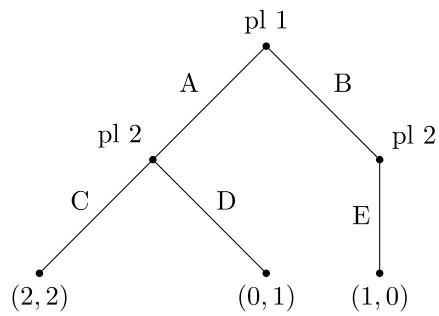
## Part II

1. (10 points) *Coordination game.* Two major smartphone manufacturers, TechNova and SmartCom, are launching new lines of smartphones. They must choose which charging standard to adopt: USB-C (widely used, fast charging) or MagSafe (Apple-style magnetic wireless charging). If both choose USB-C, the standard is universal, accessories are easy to produce, customers are happy and they both get a payoff of 2. If both choose MagSafe, it's less universal but still coordinated and both get 1 as payoff. Finally, if one chooses USB-C and the other MagSafe, consumers are confused, and third-party support is weak. Both get as payoff $-1$. Set up the corresponding bimatrix game and solve for all Nash equilibria (pure and/or mixed).
2. (10 points) Consider the normal form game

| | | **Player 2** | | | |
|---|---|---|---|---|---|
| | | W | X | Y | Z |
| **Player 1** | T | (6,6) | (1,2) | (4,4) | (8,5) |
| | B | (4,5) | (2,8) | (6,6) | (4,4) |

   (a) Which pure strategy of player 1 or player 2 is strictly dominated by a pure strategy?
   (b) Describe all combinations of strategies W and X of player 2 that strictly dominate Y.
   (c) Find all Nash equilibria of this game.

3. (20 points)
   (a) (1 point) For the game in the figure below, write down the set of players, the set of terminal histories, player function and players' preferences.
   (b) (2 points) Find the Nash equilibria of the game (after constructing the strategic form of the game in matrix format)
   (c) (2 points) Among the equilibria found previously, which one(s) is (are) subgame perfect ?



4. (20 points) Consider the Cournot duopoly model where the constant marginal cost of firm 2 is either high, $c_H$, or low, $c_L$ with $c_H > c_L > 0$. Firm 2 knows its marginal cost but firm 1 only knows that it is $c_H$ with probability $p$ and $c_L$ with probability $(1-p)$. The marginal cost for firm 1 is the constant $c > 0$. We have

   - The player set $\{1, 2\}$ ;
   - The strategy set for player 1 is $q_1 \in \mathbb{R}^+$. The strategy set for type $H$ of player 2 is $q_H \in \mathbb{R}^+$. The strategy set for type $L$ of player 2 is $q_L \in \mathbb{R}^+$.
   - The inverse demand function is $P(Q) = a - Q$ with $Q = q_1 + q_2$ and $q_2 = q_H$ or $q_2 = q_L$ depending on the realized type of player 2.

   Given these elements,

   (a) Write the payoff for player 1, denoted $\Pi_1(q_1, q_H, q_L)$, which is the expected profit realized against the two types of player 2.

   (b) Write the payoff for the two types of player 2, denoted $\Pi_H(q_1, q_H)$ for type $H$ and $\Pi_L(q_1, q_L)$ for type $L$.

   (c) Derive the three best reply functions (derivate $\Pi_i$ with respect to $q_i$ and isolate $q_i$ for $i = 1, H, L$).

   (d) Find the (Bayesian) Nash equilibrium obtained by simultaneously solving the three equations you just found.

5. (20 points) Two countries, $A$ and $B$, are in a territorial dispute over a contested island. Country $A$ does not know whether Country $B$ has high military capability (Strong) or low capability (Weak). Country $A$ assigne probability $\alpha$ to country $B$'s being strong (and $1 - \alpha$ being weak). Country $B$ knows its true type. Each country must choose between two actions : **Escalate** (analogous to Fight) and prepare for military confrontation or second action is **Back down** (analogous to Yield), avoid conflict by withdrawing claims. A country that backs down gets 0, regardless of the other country's action. If one escalates and the other backs down, the one who escalates gets 1. If both escalate and $B$ is strong, it wins the standoff: payoffs $(-1, 1)$, but if $B$ is weak, $A$ wins and the payoffs are $(1, -1)$. Formulate this situation as a Bayesian game and find its Nash equilibria if $\alpha < \frac{1}{2}$ and if $\alpha > \frac{1}{2}$.