Introduction to Game Theory

1 Mixed strategy Nash equilibrium

1. Consider the game of Chicken (depicted in the matrix below), in which two players driving their cars against each other must decide whether or not to swerve.

   (a) Is there any strictly dominated strategy for Player 1? And for Player 2?

   (b) What are the best responses for Player 1? And for Player 2?

   (c) Can you find any pure strategy Nash Equilibrium (psNE) in this game?

   (d) Find the mixed strategy Nash Equilibrium (msNE) of the game.

   (e) Show your result of part (d) by graphically representing every player i 's best response function $\text{BRF}_i(s_j)$, where $s_j = \{$ Swerve, Straight $\}$ is the strategy selected by player $j \neq i$.

		Player 2	
		Straight	**Swerve**
Player 1	**Straight**	(0, 0)	(3, 1)
	Swerve	(1, 3)	(2, 2)

2. Let us consider the following lobbying game in Figure below where two firms simultaneously and independently decide whether to lobby Congress in favor a particular bill. When both firms (none of them) lobby, congress' decisions are unaffected, implying that each firm earns a profit of 10 if none of them lobbies (-5 if both choose to lobby, respectively). If, instead, only one firm lobbies its payoff is 15 (since it is the only beneficiary of the policy), while that of the firm that did not lobby collapses to zero.

   (a) Find the pure strategy Nash equilibria of the lobbying game.

   (b) Find the mixed strategy Nash equilibrium of the lobbying game.

   (c) Graphically represent each player's best response function.

		Firm 2	
		Lobby	Not Lobby
Firm 1	Lobby	$-5, -5$	$15, 0$
	Not Lobby	$0, 15$	$10, 10$

3. Consider a variation of the above lobbying game. As depicted in the payoff matrix below, if Firm 1 lobbies but Firm 2 does not, Congress decision yields a profit of $x - 15$ for Firm 1, where $x > 25$, and does not yield any profits for Firm 2.

   (a) Find the psNE of the game

   (b) Find the msNE of the game

   (c) Given the msNE you found in part (b), what is the probability that the outcome (Lobby, Not lobby) occurs?

   (d) How does your result in part (c) varies as x increases? Interpret.

		Firm 2	
		Lobby	Not Lobby
Firm 1	Lobby	$-5, -5$	$x - 15, 0$
	Not Lobby	$0, 15$	$10, 10$

4. Consider the following Battle of the Sexes game where every player simultaneously choosing between going to the football game (F) or the opera (O). Payoffs satisfy $a_i > b_i > 0$ for every player $i = \{H, W\}$, implying that the Husband (Wife) prefers that both players are together at the football game (opera house, respectively), and attending any event is preferred to being alone anywhere. However, the premium that player i assigns to his or her most preferred event, as captured by $a_i - b_i$, can be different between husband and wife.

		Wife	
		F	O
Husband	F	a_H, b_W	0, 0
	O	0, 0	b_H, a_W

   (a) Find the best responses of each player.

   (b) How are players' randomization affected by an increase in payoffs a_H, a_W, b_H, and b_W ? Interpret.

   (c) How are your results affected if the husband earns a larger payoff from the football game than his wife earns from the opera, $a_H > a_W$? How are your findings affected otherwise?

5. Consider the following game where Player 1 has three available strategies (Top, Center, Bottom) and Player 2 has also three options (Left, Center, Right) as shown in the normal form game below. Find all Nash equilibria (NEs) of this game.

		Player 2		
		L	**C**	**R**
Player 1	**T**	3, 2	4, 3	1, 4
	C	1, 3	7, 0	2, 1
	B	2, 2	8, −5	2, 0

6. Consider the Pareto coordination game depicted in the matrix below. Determine the set of psNE and msNE , and depict both players' best response correspondences.

		Player 2	
		L	R
Player 1	U	9, 9	0, 8
	D	8, 0	7, 7

2 Extensive games with perfect information

1. Sequential Prisoner's Dilemma game.
   Consider the following Prisoner's Dilemma game where every player simultaneously chooses between confess (F) and not confess (NC). Payoffs satisfy $c > a > b > d$.

		Player 2	
		C	NC
Player 1	C	b, b	c, d
	NC	d, c	a, a

   (a) Simultaneous-move version. If players interact in a simultaneous-move game, what are their best responses? What is the NE of the game?

   (b) Sequential-move version. If player 1 acts first and, observing his action, player 2 responds, what is the SPE of the game?

   (c) Comparison. Compare the NE of the simultaneous version of the game against the SPE of the sequential version of the game. Interpret.

2. Consider the following Pareto coordination game where every firm simultaneously choosing between technology A or B, and payoffs satisfy $a > b > 0$, implying that both firms regard technology A as superior.

		Firm 2	
		Tech. A	Tech. B
Firm 1	Tech. A	a, a	0, 0
	Tech. B	0, 0	b, b

   (a) Simultaneous-move version. If firms interact in a simultaneous-move game, what are their best responses? What is the NE of the game?

   (b) Sequential-move version. If firm 1 acts first and, observing his action, firm 2 responds, what is the SPE of the game?

   (c) Comparison. Compare the NE of the simultaneous version of the game against the SPE of the sequential version of the game. Interpret.

3. More general version of the entry game. Consider the entry game in Figure below. The entrant can choose to stay out or enter into the market, and the incumbent can decide whether to accommodate or fight entry.

   (a) What will the incumbent choose in the second stage of the game?

   (b) What will the entrant do in the first stage of the game?

   (c) Characterize the SPEs of this game. Interpret.