General Equilibrium Theory — Elena del Mercato

Part 1's Final Exam - 2 hours — **December 15, 2025 - Master M2 APE**

Do the best you can in the time allotted. Read and think before you write, and try to be both concise and precise.

Exercise 1. Consider a classical exchange economy with L commodities. The set of individuals is $\mathcal{I} = \{1, ..., i, ..., I\}$. For every individual $i \in \mathcal{I}$ the consumption set is $X_i = \mathbb{R}^L_+$, the individual endowment is $e_i \in \mathbb{R}^L_+$, and the utility function u_i is continuous, quasi-concave, and locally non-satiated on X_i. The set of prices is $P = \{p \in \mathbb{R}^L : \|p\| \leq 1\}$, and the set of feasible allocations is $F = \left\{x = (x_1, ..., x_i, ..., x_I) \in \mathbb{R}^{LI}_+ : \sum_{i\in\mathcal{I}} x_i \leq \sum_{i\in\mathcal{I}} e_i\right\}$.

1. Recall the definitions of the budget correspondence $\widetilde{\gamma}_i$, the quasi-budget correspondence $\widetilde{\delta}_i$, and the quasi-demand correspondence $\widetilde{\varphi}_i$, where i's individual wealth $p \cdot e_i$ has been increased by $\dfrac{(1 - \|p\|)}{I}$.
2. Prove that the correspondence $\widetilde{\varphi}_i$ is upper semicontinuous on P.
3. Recall the definition of the Walrasian Auctioneer correspondence ψ.
4. Consider the correspondence $\widetilde{\theta}(p, x) = \psi(x) \times \prod_{i\in\mathcal{I}} \widetilde{\varphi}_i(p)$. Let $(p^*, (x_1^*, ..., x_i^*, ..., x_I^*))$ be a fixed point of the correspondence $\widetilde{\theta}$. Prove that the three following properties hold true.

   (a) If $p^* \cdot \sum_{i\in\mathcal{I}} (x_i^* - e_i) > 0$, then $\|p^*\| = 1$.

   (b) $p^* \cdot \sum_{i\in\mathcal{I}} (x_i^* - e_i) = 0$.

   (c) $\sum_{i\in\mathcal{I}} (x_i^* - e_i) = 0$.

Exercise 2. There are two individuals and two commodities. The consumption sets are the open sets $X_1 = X_2 = \mathbb{R}^2_{++}$, and the initial endowments are $e_1 = (2, 2)$ et $e_2 = (3, 3)$. The generic consumption of individual i is $x_i = (x_{i1}, x_{i2})$. Individual 2 suffers of a positive externality, because of the consumption of commodity 1 by individual 1. The utility functions are:

$$u_1(x_1) = x_{11} + \ln x_{12} \quad \text{and} \quad u_2(x_2, x_{11}) = x_{21} + \ln x_{22} + \frac{1}{2} x_{11}$$

1. Recall the definition of a competitive equilibrium (à la Nash), and determine the individual marginal rates of substitution at equilibrium.
2. Recall the definition of a Pareto optimal allocation.
3. Determine the first order conditions (FOC) associated with Pareto optimality.
4. Show that a competitive equilibrium allocation is not Pareto optimal.