Competitive Equilibrium
and Market Failures

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This booklet brings together all the tutorial problem statements corresponding to the first six lectures of the course. These lectures cover the relationship between equilibrium and the optimum in partial equilibrium, first under perfect competition and then in the presence of some market failures (externalities and public goods).

In some cases, several problems are associated with the same tutorial. The choice of the problem to be discussed is left to your tutorial instructor; the remaining problems are provided as additional practice exercises. Several of the problem statements are adapted from questions used in previous midterm examinations.

Tutorials will begin during the week of 28 September 2026; ten 1.5-hour sessions are planned.

TD1. Firm Supply in Partial Equilibrium

Consider a firm j with cost function

$$c_j(y) = a_j y + \frac{1}{2} b_j y^2$$

for producing $y \geq 0$ units of a final good, where $a_j > 0$ and $b_j > 0$ are two constants.

1. Show that the cost function is increasing and convex.

2. Plot this function in a diagram with y on the horizontal axis and monetary amounts, measured in euros, on the vertical axis.

3. Express the change in cost implied by producing $dy > 0$ additional units, where dy is close to zero, as a function of y, the parameters a_j and b_j, and dy. What is this change if the firm was producing $y = 100$ units of the final good when it decided to increase production by $dy = 1$ unit? Illustrate graphically the consequences of this change in output for cost.

4. Let $p > 0$ be the price of the final good. What is firm j's supply $y_j(p)$?

5. Suppose the price changes by a small amount, from p_0 to $p_1 = p_0 + dp$, with $dp > 0$. Express the response of the firm's supply as a function of b_j and dp.

6. Illustrate graphically the impact of the price change on firm j's supply when $a_j < p$.

7. What is the maximum profit $\pi_j(p)$ that this firm obtains at price p?

8. Let $p_0 > a_j$. Express the response of profit to the price change from p_0 to p_1 as a function of p_0, dp, and the two parameters a_j and b_j. Illustrate this perturbation graphically.

9. When J firms are present in the market, are they all active? Suppose $a_j = j$ for every $j = 1, \ldots, J$. What is the aggregate supply function $Y(p)$?

TD2. Equity and Efficiency of Support for Firms in Difficulty

Consider a market with two firms, $j = 1, 2$, behaving competitively. Their technologies are characterized by the cost functions $c_j(y)$, with

$$c_1(y) = \frac{y^2}{2}, \qquad c_2(y) = y + \frac{y^2}{2},$$

where y is the number of units produced. For simplicity, assume that there is a single consumer, who owns the firms and initially has income $\bar{m}$. The consumer's preferences are represented by the utility function

$$u(x) + m = 2\sqrt{x} + m$$

when consuming x units of the good and retaining m euros (which can be spent on other goods).

1. **Supply**

   (a) Show that firm 1 is active at every positive price, whereas firm 2 is inactive when $p < 1$.

   (b) What is firm j's supply function $y_j(p)$?

   (c) What is aggregate market supply $Y(p) = y_1(p) + y_2(p)$? Plot it in a diagram with quantities on the horizontal axis and prices on the vertical axis.

2. **Demand**

   (a) Write the consumer's budget constraint. Using this constraint, write the consumer's utility without referring explicitly to money holdings m.

   (b) By maximizing the utility obtained in the previous question, show that aggregate demand is
   $$X(p) = \frac{1}{p^2}.$$
   Plot it in a diagram with quantities on the horizontal axis and prices on the vertical axis.

3. **Equilibrium**

   (a) Let p^* be the equilibrium price. What equation determines this price if $p^* < 1$? How does the equation change if $p^* \geq 1$?

   (b) Show that $p^* = 1$. What is each firm's output in equilibrium?

4. **The optimum**

   We now consider the optimality properties of the equilibrium: what recommendations about firms' activity follow from applying a Marshallian criterion?

   (a) Compute the household's gross surplus and net surplus if it consumes x units of the good. Illustrate them graphically with reference to marginal utility.

   (b) Suppose the public authority wants the firms to produce a total quantity y of the good. How should this quantity be allocated between the two firms?

   (c) Using the result from question 4(b), write the production cost $c(y)$ of y units of the good.

(d) Write the program that determines the Marshallian optima and characterize these optima. Comment.

5. **Optional – to be done after Chapter 3.** Firm 2's activity is part of an industrial policy intended to preserve the firm. This policy must apply to the entire sector. A subsidy s is therefore paid to all producers: when consumers pay price q for the good, producers receive $p = q + s$ euros per unit.

   (a) Express the new aggregate supply function as a function of the price q paid by consumers.

   (b) Plot the new supply function and the pre-subsidy supply function in a diagram with quantities on the horizontal axis and price q on the vertical axis. How does the equilibrium price received by producers change when the subsidy is introduced?

   (c) Write the market-clearing equation equating supply and demand when $s > 0$.

   (d) Express the price change dq^* as a function of a change in the subsidy ds for a small subsidy ($s \simeq 0$). Show that, for $q^* = 1$ and $s = 0$, the change is

$$dq^* = -\frac{1}{2}\, ds.$$

   (e) By how much does the price received by producers change?

   (f) Compute the Harberger deadweight loss. Can one say that the subsidy policy is more desirable the more it benefits consumers?

TD2. Ursula and the Electric Fairy Tales

In a note published last September[1], the European Commission

0.3 proposes a temporary revenue cap for "inframarginal" electricity producers, namely producers using lower-cost technologies such as renewables, nuclear power and lignite, which supply electricity to the grid at a cost below the price level set by more expensive "marginal" producers. These inframarginal producers have earned exceptional revenues while their operating costs have remained relatively stable, because expensive gas-fired power plants have driven up the wholesale electricity price they receive. The Commission proposes setting the cap on inframarginal revenues at 180 euros/MWh. This will allow producers to cover their investment and operating costs without jeopardizing investment in new capacity, in line with our 2030 and 2050 energy and climate objectives. Revenues above the cap will be collected by Member State governments and used to help energy consumers reduce their bills.

In the electricity market, consumers demand $X(p) = 20 - p$ megawatt-hours (MWh) when the unit price is p euros. There are two different firms. Firm 1 produces electricity using nuclear power. It must spend $y^2/2$ euros to produce $y \geq 0$ MWh. Firm 2 uses gas-fired thermal generation and has production cost $y + y^2/2$ euros for y MWh.

I. Equilibrium

1. (a) What is firm 1's supply $y_1(p)$ at price p? (b) Show that firm 2 prefers to remain inactive when $p \leq 1$. (c) Deduce firm 2's supply $y_2(p)$ at price p.

2. Write aggregate supply $Y(p)$ as a function of price p.

3. What would the equilibrium price be if firm 1 were the only firm producing electricity? Show that firm 2 would prefer to be active at this price. What do you conclude?

4. What would the equilibrium price be if both firms produced electricity? Plot aggregate supply, demand, and the equilibrium price in a diagram with the number of MWh on the horizontal axis.

5. Compute each firm's profit and consumer surplus in equilibrium. Why does firm 1 always earn higher profits than firm 2?

II. Distributional effects of an increase in costs

A conflict involving a major gas producer raises firm 2's production cost, which becomes $c_1 y + y^2/2$ euros.

1. Let $c_1 = 4$ euros. What happens in the market? How are firms' profits and consumer surplus affected?

2. Repeat the previous question for a larger increase in cost: $c_1 = 16$ euros.

3. Can one say that the nuclear electricity producer benefits from an increase in the cost of gas-fired electricity generation?

III. Policies to limit profits

From now on, assume $c_1 = 4$. The differences highlighted in the previous part lead the public authority to look for ways to limit firms' revenues. It considers two ways of introducing a price cap, denoted s, with $s > c_1$.

[1] European Commission, *Energy prices: Commission proposes emergency market intervention to reduce bills for Europeans*, Press release, Brussels, 14 September 2022. Available at `https://france.representation.ec.europa.eu/informations`.

III. A. Price cap

A firm selling y MWh at price p must pay a tax of $p-s$ euros per unit produced when $p \geq s$. It pays no tax when $p < s$.

1. Write each firm's after-tax profit when the public authority chooses a cap s below the market price, $s \leq p$.

2. Compute each firm's supply and aggregate supply at price p when $s \leq p$.

3. What is the equilibrium price p^* when the public authority actually levies taxes on firms ($s \leq p^*$)?

4. Assume $s \leq p^*$. Show that consumer surplus evaluated at price p^* falls when the public authority lowers the cap s imposed on firms. Interpret this result.

5. How does consumer surplus vary with the cap s if the public authority rebates to consumers the tax revenue collected from firms? Do you think this is the outcome sought by the public authority? Explain.

III. B. Lump-sum taxes

The results above lead the public authority to consider another tax policy involving a cap on the price received by firms.

1. Suppose firm j must pay a lump-sum tax of T_j euros. What is firm 1's supply? What is firm 2's supply?

2. The authority wants to ensure that no firm makes a loss. What is the equilibrium price? Compute each firm's profit as a function of T_1 and T_2.

3. The authority chooses the tax amount as

$$T_j = \begin{cases} (p^* - s)y_j(p^*) & \text{if } p^* \geq s, \\ 0 & \text{otherwise.} \end{cases}$$

   where p^* is the equilibrium price and $y_j(p^*)$ is firm j's supply at that price. Can the public authority confiscate all of firm 1's profit while ensuring the viability of firm 2?

4. The public authority gives up on keeping firm 2 in the market. What value of the price cap s makes it possible to confiscate all of firm 1's profit?

5. Conclude on the trade-off between taxing inframarginal firms and consumer welfare that the public authority must resolve.

TD3. Incidence of Housing Assistance

On the website `www.capital.fr`, Thomas Chenel commented on 12 July 2019 on Véronique Flambard's synthesis[2] of research on housing assistance:

> On the more general question of reducing housing allowances, the author describes it as a political decision. "The cost of housing allowances has increased considerably, year after year (around 18 billion euros of public expenditure in 2017, according to the study, editor's note). The government's primary motivation is to control this cost. It is a budgetary necessity."
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> According to her, it is also possible that, by reducing assistance, the government hopes to reduce rent inflation. The study emphasizes that the broad coverage of housing allowances has contributed to rent increases, since landlords can request that housing benefits be paid directly to them on behalf of their tenants in order to avoid rent arrears. This mechanism encouraged many landlords to inflate their rents artificially, as a 2006 study by Gabrielle Fack, professor of economics at Université Paris Dauphine, had already shown. Using the housing-benefit reform of the early 1990s, Fack estimated that one additional euro of housing assistance translated into a 78-cent increase in rent, leaving only 22 cents to the beneficiary to reduce the burden of housing costs.

To reconstruct the quantitative exercise carried out by Gabrielle Fack, consider a partial-equilibrium market in which supply and demand at price p are

$$O(p) = p^{\sigma} \qquad \text{and} \qquad D(p) = p^{-\delta},$$

with $\sigma > 0$ and $\delta > 0$. Initially, the good is not subsidized and the equilibrium price is denoted p_0^*. The government introduces an ad valorem subsidy at rate s benefiting consumers: when suppliers sell the good at price p, buyers actually pay $(1-s)p$.

1. Interpret the parameters σ and δ.

2. In a diagram with p on the vertical axis (and quantities on the horizontal axis), plot the supply function and the two demand functions for $s = 0$ and for some $s > 0$ (for example, $s = 20\%$). Mark the initial equilibrium E_0 on the graph, and likewise the final equilibrium E_1. How does the equilibrium price received by suppliers change? How does the price paid by buyers change?

3. Identify on the graph consumer surplus, producer surplus, and the government's surplus. How does total surplus change? Using only the change in surplus as the criterion, does the subsidy policy generate a social loss or gain?

4. What relationship is satisfied by the new equilibrium price p_1^* received by suppliers? Express this price as a function of the subsidy rate s and the ratio σ/δ.

5. For which values of σ/δ does the producer price remain unchanged? Interpret.

6. For which values of σ/δ does the consumer price remain unchanged? Interpret.

7. We now consider the housing market. The table below reports Fack's (2005) estimates of the effect of housing assistance on rents (the price of housing services). To interpret it, assume that tenants in the "second quartile" receive almost no housing assistance. What value would you choose for the ratio σ/δ? Conclude on the difficulty, in a market economy, of helping poor households afford housing.

[2] Véronique Flambard, 2019, *Les allocations logement ne peuvent à elles seules empêcher les arriérés de loyer* [Housing allowances alone cannot prevent rent arrears], Économie et Statistique 507–508.

Table 1
A first estimate of the effect of housing assistance using the difference-in-differences estimator

		1988	1996	Change 1988 to 1996
Annual rent per m^2 (2002 euros)	1st quartile	64.1 (2.2)	90.2 (2.5)	26.1 (2.3)
	2nd quartile	67.8 (2.2)	79.0 (2.2)	11.2 (2.2)
	Difference	−3.7 (2.4)	11.2 (2.1)	**14.9** (3.2)
Annual housing assistance per m^2 (2002 euros)	1st quartile	14.3 (0.5)	34.9 (1.0)	20.6 (1.4)
	2nd quartile	7.5 (0.2)	12.3 (0.3)	4.8 (0.8)
	Difference	6.8 (1.0)	22.6 (1.2)	**15.8** (1.5)
Wald estimator (14.9/15.8)				0.94 (0.20)

Reading note: the change in rent per m^2 between 1988 and 1996 is 26.1 euros for the first quartile and 11.2 euros for the second quartile. Rent per m^2 in the first quartile therefore rose by 14.9 euros more than in the second quartile between 1988 and 1996. Housing assistance per m^2 in the first quartile rose by 15.8 euros more than in the second quartile over the same period. Thus, one additional euro of assistance leads to a 0.94-euro increase in rent.
Standard errors are in parentheses.
Sample: tenant households in the private sector.
Source: author's calculations based on the Insee housing surveys.

TD4. Bargaining over Pollution

Wood heating generates toxic air pollutants that move with the wind. Consider two consumers, $i = 1, 2$. Consumer 1's preferences are represented by the utility function $\ln x + m$ when the consumer uses x steres of firewood and spends m euros on other goods. Agent 2's preferences are represented by $-\alpha(x - E)^2 + m$, where E is an externality. This externality is caused by agent 1's wood consumption, $E = ax_1$. Each agent has income $\bar{m}$ euros, assumed to be very large relative to spending on wood. The cost of producing y steres is cy euros.

Part 1. Equilibrium

1. Compute the marginal utility obtained by consumer 2 after consuming x steres of wood. At what number of steres can one say that one additional stere no longer provides utility to this consumer? In what follows, restrict attention to consumption levels below this amount.
2. What is the aggregate demand function for steres of wood?
3. What is the aggregate supply of steres?
4. Compute the Walrasian equilibrium price and the quantities consumed by each of the two agents.

Part 2. Optimum

1. Write Marshallian surplus.
2. Compute the quantities that should be consumed by each of the two agents at a Marshallian optimum.
3. How does agent 1's optimal consumption differ from the agent's equilibrium consumption? Explain.
4. In what sense can one say that agent 2 should consume smaller quantities, but more often?

Part 3. Coasian bargaining

Suppose agent 2 is endowed with a right that allows the agent to limit agent 1's wood consumption to a single stere. Agent 2 proposes that agent 1 be allowed to consume x_1 steres in exchange for paying agent 2 compensation of T euros. The pair (x_1, T) offered to agent 1 is chosen by agent 2. If agent 1 accepts the proposal, the agent must purchase quantity x_1 on the market.

1. What is agent 1's utility if the agent rejects the option (x_1, T) proposed by agent 2?
2. What is agent 1's utility if the agent accepts it?
3. Write the program agent 2 must solve to choose optimally the agent's own consumption x_2 and the offer (x_1, T) to make to agent 1, if agent 2 wants agent 1 to accept the proposal.
4. Express the transfer T that agent 2 should propose as a function of x_1 and $\bar{x}_1$.
5. Rewrite agent 2's objective as a function of x_2 and x_1. What do you observe? Deduce the wood-consumption levels that this agent should choose. Comment.

TD4. Towards Cleaner Agriculture?

Motivated by the recent debate over anti-pesticide regulations recommending a minimum distance between treated fields and homes or schools, consider a small grower who treats a vineyard located near an urban area that is the main outlet for the wine produced. The winegrower could reduce the pollution emitted by using a costly enclosed-spraying technique. We say that the winegrower abates pollution. This exercise studies the optimal abatement policy and the instruments that the public authority could use to induce the winegrower to improve environmental quality.

When the winegrower produces y litres of wine, emissions are

$$e = \begin{cases} y - a & \text{if } y - a \geq 0, \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where a is interpreted as pollution abatement. The cost of producing y and abating a units of pollution is

$$\frac{y^2}{2} + \phi \frac{a^2}{2}$$

euros, with $\phi \geq 1$.

The household (representative of households in the area) obtains utility $2 \log x + m - 2e$ when consuming x litres of wine and keeping m euros to buy other goods. Its income is $\bar{m}$ euros. The harm it suffers, equal to $2e$, is treated as an externality.

I. Equilibrium

Let p be the price of one litre of wine.

1. What is the winegrower's profit when selling y litres of wine and abating pollution by an amount a?
2. What is the supply function for wine?
3. Why does the winegrower choose not to abate pollution?
4. Write the household's budget constraint.
5. What is the household's demand function for wine?
6. What are the equilibrium quantity and the equilibrium price of one litre of wine?
7. What is equilibrium pollution?

II. The optimum

1. Why is the total amount of money required to produce y litres of wine, abate pollution by an amount a, and allocate m euros to the household equal to

$$\frac{y^2}{2} + \phi \frac{a^2}{2} + m\,?$$

2. Write the feasibility constraint requiring the total amount of money used by the planner to be no greater than the total amount of money available.
3. Write the feasibility constraint requiring total wine consumption to be no greater than wine production.

4. Write the planner's problem of maximizing household utility subject to the feasibility constraints obtained in questions II.2 and II.3 while taking into account the relationship between pollution, production, and abatement described by (2).

5. Why will the planner always choose to make both feasibility constraints bind?

6. Using the result from question II.5, rewrite the planner's objective without referring to output y or money holdings m.

7. By differentiating the objective obtained in question II.6, find the optimal wine output and the amount of pollution abatement chosen by the planner.

III. Inefficiency of equilibrium

1. Using the results from question II.7, compute optimal pollution e^*. Should pollution be eliminated completely?

2. What is household utility at the optimum?

3. Is equilibrium pollution too high?

4. Assuming that the winegrower's profits are transferred to the household, compare household utility in equilibrium with the utility it could attain at the optimum.

IV. On the polluter-pays principle

1. First apply the polluter-pays principle by requiring the winegrower to pay a tax t per litre of wine. With this tax alone, does the winegrower behave optimally?

2. Now supplement the tax described in question IV.1 with a subsidy s for each unit of pollution abated. What is the new wine-supply function? Show that the amount of abatement chosen by the winegrower is s/ϕ. Comment.

3. What tax t and subsidy s should be chosen for the equilibrium to coincide with the optimum?

4. What is the total net revenue collected by the tax authority for the values of t and s obtained in question IV.3? Would the tax on wine be sufficient to finance the abatement subsidy policy?

For additional practice, one possible variant is to interpret the parameter ϕ as the effectiveness of abatement: the externality would then be $y - \phi a$ (when this amount is positive). For simplicity, the parameter would then be removed from the cost function.

TD5. A Swimming Pool Too Good for You

> The département 93 also has its villages. In Coubron, Vaujours and Gournay-sur-Marne, located on the border with Seine-et-Marne, the population does not exceed 7,000. The setting is rural, without large housing estates or excessive urban development. These small towns resemble municipalities in the outer suburbs or peripheral France. Their problems do too. With tight budgets, elected officials "struggle every day," in the words of Eric Schlegel, mayor (SE) of Gournay, to maintain public services. "Just because 60% of our area is green space does not mean our residents are not entitled to high-quality infrastructure," says Ludovic Toro, mayor (UDI) of Coubron. "So we have become experts at hunting for subsidies!"
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> Without subsidies, there would be no multimedia library. Ten days ago, the mayor signed a local development agreement with the Île-de-France region. It will make it possible to build a leisure centre and a multimedia library. "Regional subsidies account for 50% of the budget for the two projects. Without them, we could not carry them out," says Ludovic Toro. "We still have to find other forms of support, because I want only 30% of the cost to remain with the municipality."
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> In Gournay, Eric Schlegel managed to persuade the prefecture and obtained 12,000 euros in subsidies to install video surveillance. "After a very long battle," the mayor says. "The situation is becoming untenable. We do not have the means to develop our town, while the state asks us to build housing. But what if we cannot provide the public services that go with it?" The mayor "does not know" how he will finance the construction of a third school complex, which will be essential to educate the children of new residents in the coming years. "It would be easier if we were a large city," Eric Schlegel says. "I am not saying they are better off, but to obtain subsidies you first need the resources required to plan an investment. It is a vicious circle!"

The struggle of Seine-Saint-Denis's small towns to maintain public services (excerpts)
Victor Tassel
Le Parisien, 18 September 2019

Consider a society made up of n identical agents. The agents are divided into two different groups, $j = 1, 2$. Group j contains n^j agents (so $n^1 + n^2 = n$). Each agent has preferences over a local public good and money spent on other goods. A local public good is a good that benefits only members of the group: think, for example, of infrastructure such as a swimming pool, library, stadium, etc., benefiting only residents of municipality j. The preferences of a member of group j are represented by the utility function $\log G^j + m$, where G^j is the quantity of a local public good and m is the amount of money. Each individual has income $\bar{m}$ euros. The total amount of money available is fixed and equal to $\bar{M}$.

1. Does this framework seem to provide a good representation of the situation of the "villages of département 93"?

2. Express the Pareto-optimal quantities of public goods as a function of n^1 and n^2. How are the recommendations implied by the Samuelson rule expressed?

3. Suppose the central government can require each member of group j to pay a group-specific contribution τ^j. What contribution should be chosen so that each agent voluntarily demands the Pareto-optimal quantity for the agent's own group?

4. In practice, it can be difficult to vary taxes across groups: the same rule is applied uniformly to everyone. Now impose $\tau^j = \tau$ for all $j = 1, 2$. What amount τ must each person pay for G^1 and G^2 public goods to be provided in groups 1 and 2? What quantities of public goods do group members prefer under such a tax? Are they optimal? Comment.

5. (Optional question). Some local public services in fact benefit several neighboring small municipalities; school or sports infrastructure are examples. To account for this, suppose a member of group i benefits from a fraction α of the public good of group j, $j \neq i$. Write the utility of a member of group i. Characterize the Pareto optima. Do there exist urban sizes n^1 and n^2 and a value of α for which the tax regime leads to an optimal quantity of public goods? Comment in light of intermunicipal cooperation policy.

TD5. Drought and Water Conflicts

In France, the Prefecture reserves a quota of water in groundwater aquifers for biodiversity. The drought at the beginning of 2023 intensified tensions between agricultural needs and biodiversity preservation, with farmers seeking to draw on the quota reserved for biodiversity in order to irrigate crops. This exercise studies the optimal management of groundwater. There are two agents, $i = 1, 2$. Agent i has utility

$$m - \frac{1}{2}(\bar{e}_i - e)^2$$

if the agent spends m euros on consumption and the water-table level is e metres. The agent has given income $\bar{m}_i$ (denote by $\bar{M}$ the total amount of money $\bar{m}_1 + \bar{m}_2$). Assume $0 < \bar{e}_1 < \bar{e}_2 < \bar{e}$, where $\bar{e}$ is the maximum water-table level. Assume also that $2\bar{e}_1 > \bar{e}$.

Part I. The optimal water table

1. What water-table level is preferred by agent i? Which agent better represents farmers?
2. Starting from e, with $e < \bar{e}_i$, the water table rises by a small amount de, $de > 0$. Express, as a function of e and $\bar{e}_i$, the change dm_i in agent i's consumption expenditure that would leave utility unchanged. Would this agent accept a reduction in expenditure?
3. Would agent i accept a reduction in expenditure if initially $e > \bar{e}_i$? Interpret with reference to the marginal rate of substitution.
4. For each of the two agents, draw in a diagram with the water-table level on the horizontal axis and consumption expenditure on the vertical axis the indifference curve that gives agent i the greatest welfare. Which water-table levels generate a conflict between farmers and environmentalists defending biodiversity?
5. The Prefecture, guided by Marshallian principles, seeks to maximize the sum of the two agents' utilities. Show that it chooses an optimal water-table level e^* satisfying the Samuelson rule. Express e^* as a function of $\bar{e}_1$ and $\bar{e}_2$. Is the Prefecture led to intensify the conflict between the agents?

Part II. Delegating groundwater management

1. Management of the water table is delegated to the environmentalists. They can offer farmers a contract specifying a water-table level e in exchange for a payment of T euros.
   (a) Write the farmers' utility if they accept the contract.
   (b) What water-table level is chosen if they reject it? Write the farmers' utility in that case.
   (c) What is the best contract acceptable to farmers that the environmentalists can offer? Compare the chosen water-table level with e^*.
   (d) Do the environmentalists benefit from offering a contract to the farmers?
2. Management is now delegated to the farmers. They can offer a contract (e, T) in which water-table level e is associated with a transfer they receive from the environmentalists.
   (a) What is the best acceptable contract offered by the farmers? What classic microeconomic result does this remind you of?
   (b) Do the farmers gain by offering a contract to the users?

3. Show that the environmentalists would prefer to manage the water table. In that case, draw each agent's indifference curves in the same diagram as in I.4.

4. In view of the distributional problems suggested by question II.3, management is delegated to a private firm that seeks to maximize its revenue. It offers agent i water-table level e in exchange for payment T_i ($i = 1, 2$). It sets the water table at level $\hat{e}$ as soon as one agent refuses to subscribe to the contract. Show that it chooses $e = e^*$. What level $\hat{e}$ would be chosen in the event of non-subscription? How do the firm's revenues vary with $\bar{e}_1$ and $\bar{e}_2$? Explain.

5. In practice, some farmers care about biodiversity, and the water-table level that preserves biodiversity remains difficult to assess. Implementing contracts with different payments across agents is therefore debatable. Repeat question II.4 when $T_1 = T_2 = T$. Does the firm still propose the level e^*? Comment.