Chapter 3 Tax incidence

I lump-sum personalized taxes / transfers

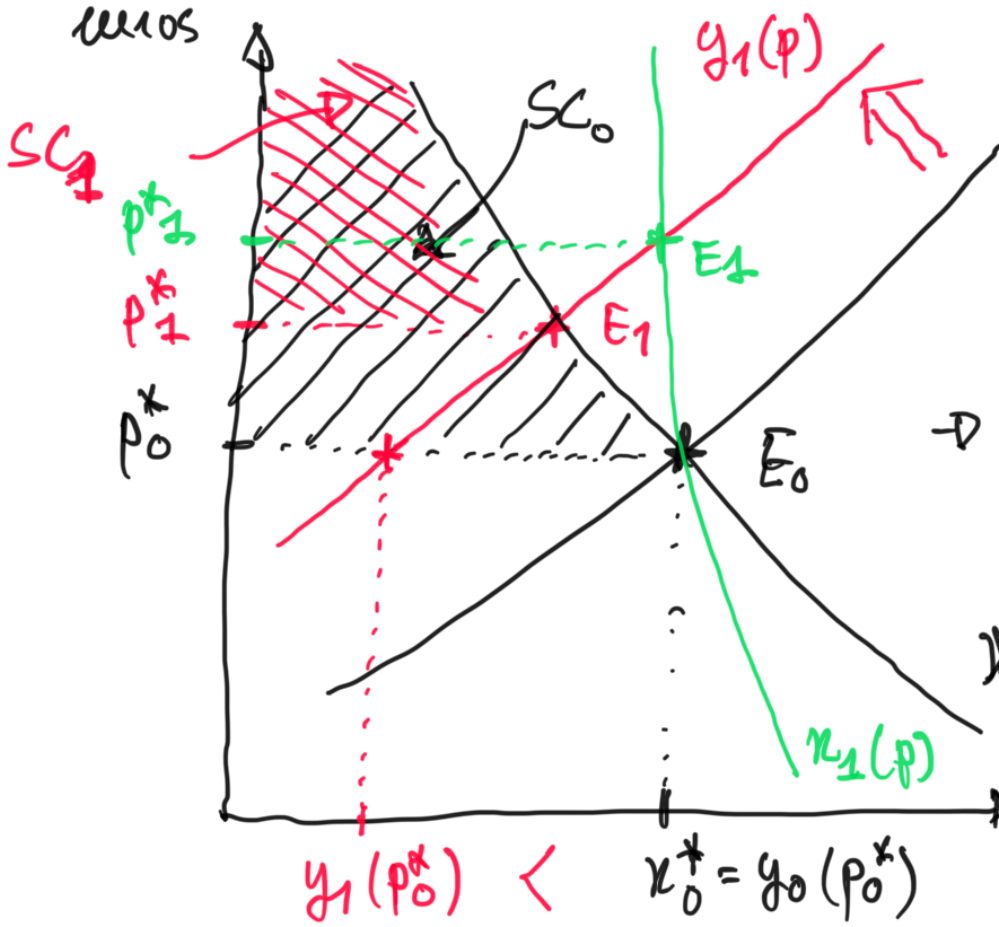
I two specific examples

Example 1 $C(y) = \frac{1}{2} a y^2$

$$a = \begin{cases} a_0 & \text{initially} \\ a_1 & \text{finally} \end{cases} \qquad a_1 > a_0$$

$$y(p) \;/\; C'(y(p)) = p \iff a\, y(p) = p \iff y(p) = \frac{p}{a}$$

$$y_0(p) = \frac{p}{a_0} \;\to\; y_1(p) = \frac{p}{a_1} < \frac{p}{a_0} = y_0(p) \;\forall p$$



$\pi_0 \not\leq \pi_1$
Producers gain from the tax since $p_0^* \to p_1^*$

$SC_0 > SC_1$

Example 2 $u(x) = b \ln x$

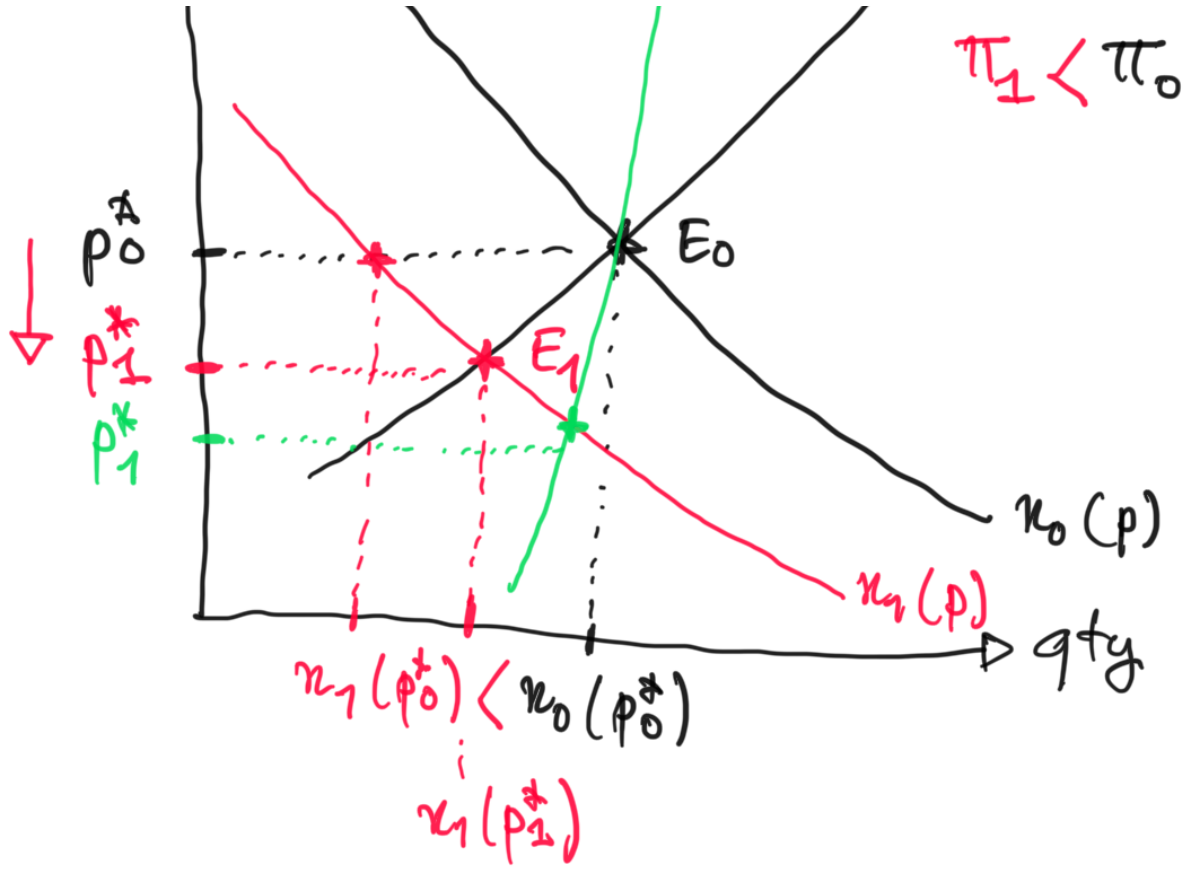
$$b = \begin{cases} b_0 & \text{initially} \\ b_1 < b_0 & \text{finally} \end{cases}$$

$$x(p) \;/\; u'(x(p)) = p \iff \frac{b}{x(p)} = p$$

$$\iff x(p) = \frac{b}{p}$$

$$x_0(p) = \frac{b_0}{p} \to x_1(p) = \frac{b_1}{p} < \frac{b_0}{p} = x_0(p)$$





→ The side of the market that is the most insensitive to price changes is the one that is hurt by exogenous shocks, whether these shocks affect producers or consumers.

II taxation of consumption

p price that is received by producer
"producer price"
t tax (unit tax) in euros (excise tax)

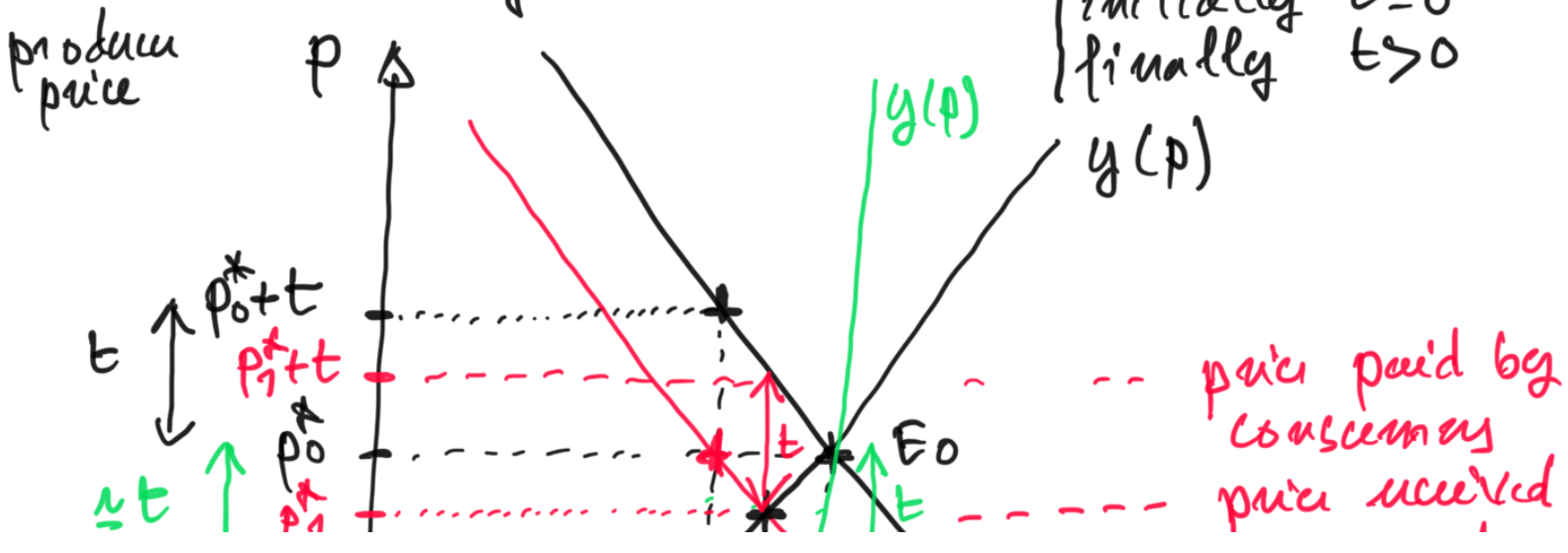
$q = p + t$ $\quad (t = \tau p)$

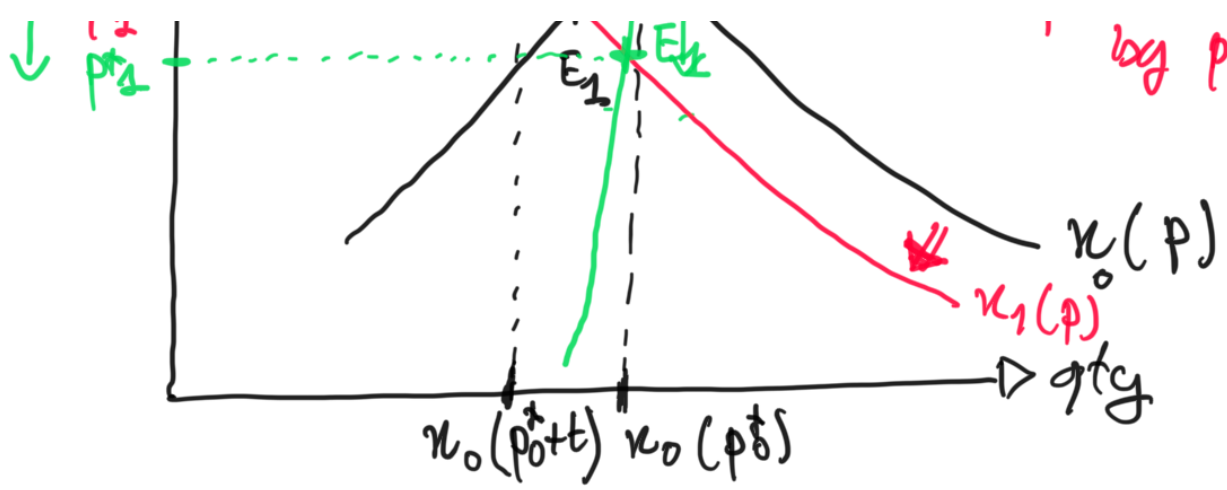
↳ paid by consumers
"consumer price"

↑↑ net-of-tax price
tax rate

II.A. Graphical representations of the impact of introducing the tax t.

initially $t = 0$
finally $t > 0$





log producers

$p_0^* > p_1^* \Rightarrow$ producers get lower profits $\Pi_0 > \Pi_1$

$p_0^* < p_1^* + t \Rightarrow$ consumers get lower surplus

$CS_0 > CS_1$

$p_1^* - p_0^* \leq -t$

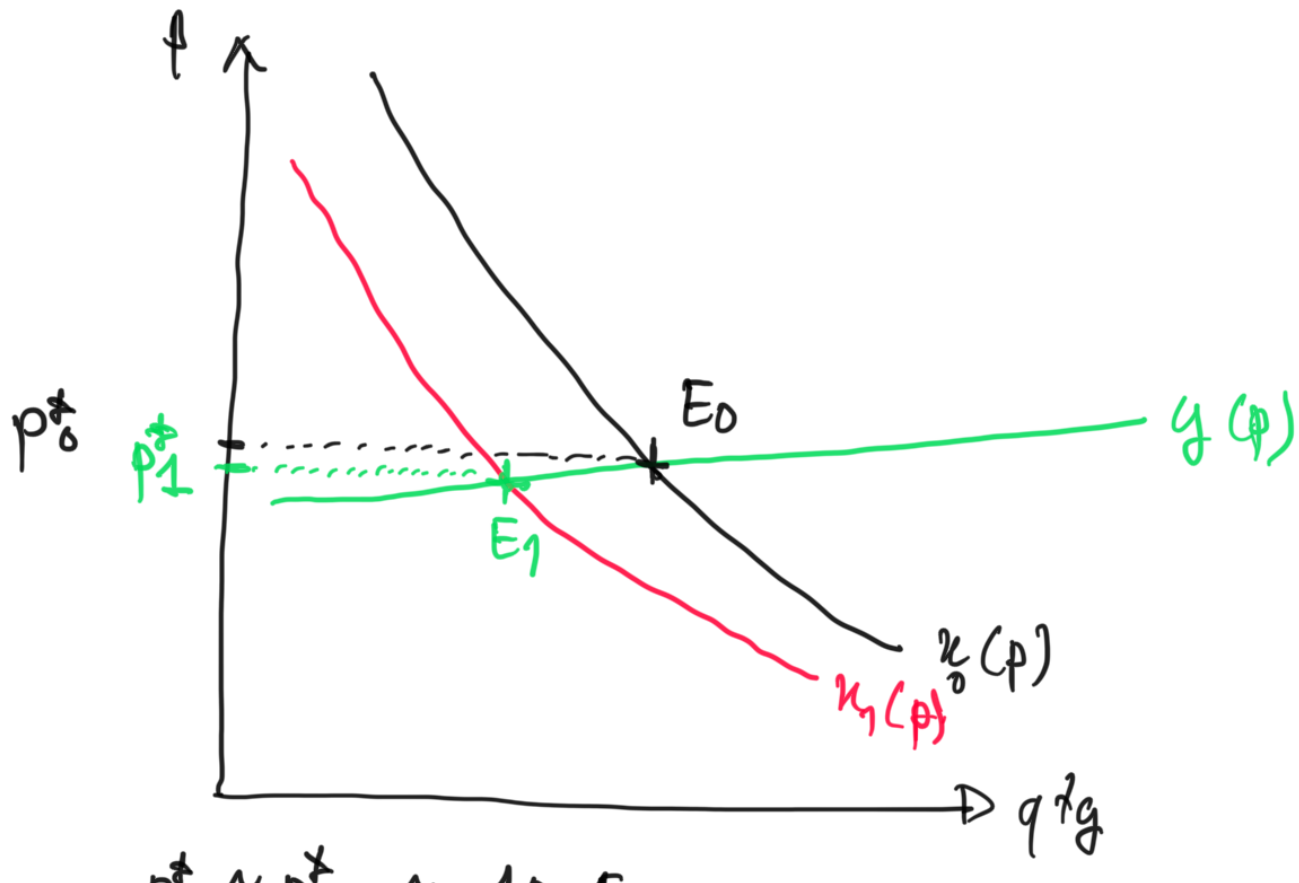
suppose $p_0^* = 10$ € , $t = 1$ €

$q_1^* = p_0^* + t = 11$ €

In the given supply curve

$p_1^* = 9$ € , $t = 1$

$q_1^* \leq 10$ €



$p_0^* \simeq p_1^* \simeq 10$ €

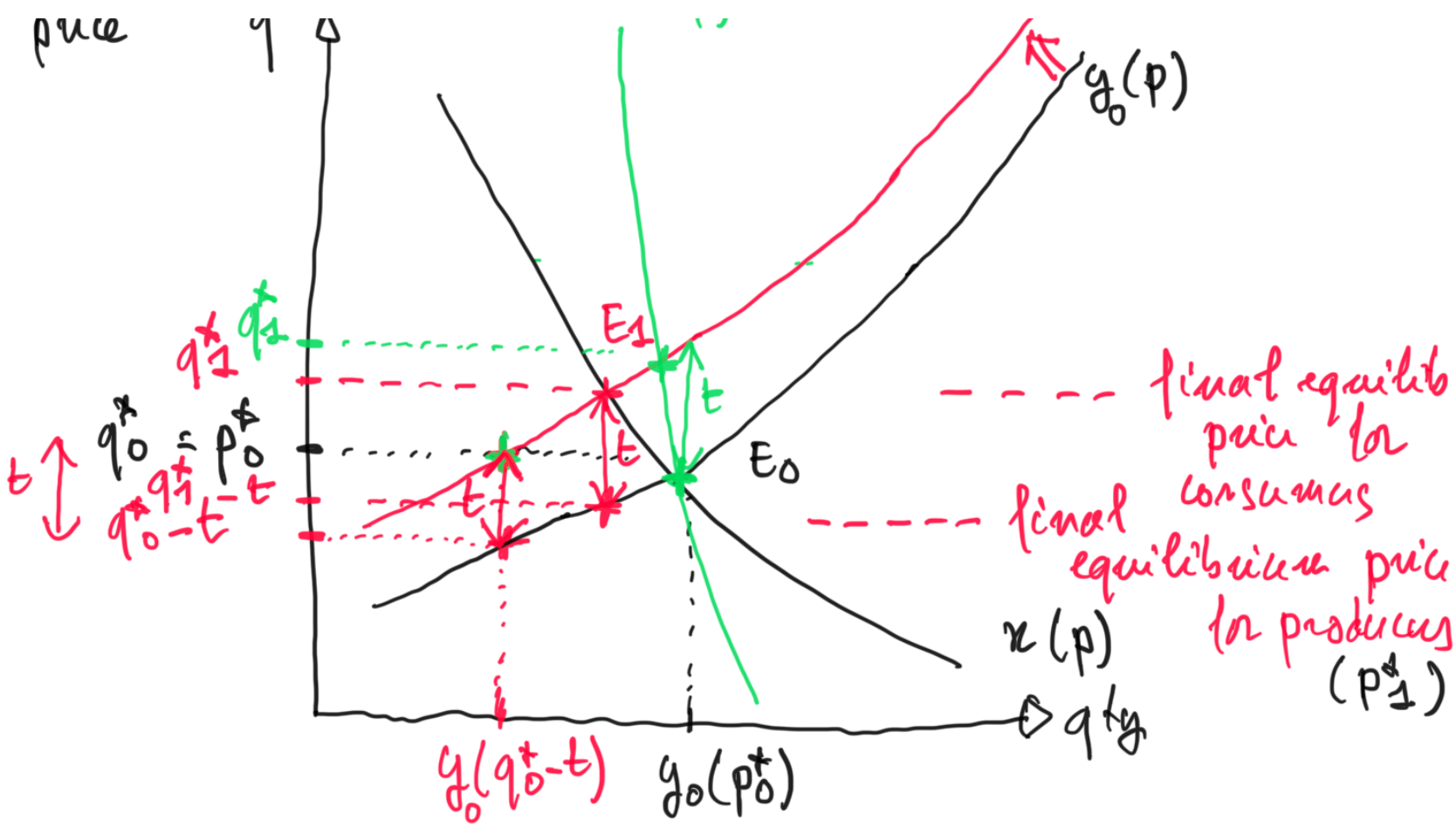
$q_0^* = p_0^*$

$q_1^* = p_1^* + t \simeq p_0^* + t = 10 + 1 = 11$ €

consumers

$x(p)$

$y_1(p)$



1) $q_1^* > q_0^*$ ↑ price for consumers
consumers loose $CS_0 > CS_1$

2) $q_0^* = p_0^* > q_1^* - t = p_1^*$
↓ price for producers
producers loose $\pi_0 > \pi_1$

$q_0^* \to q_1^* \simeq q_0^* + t$
$p_0^* \simeq p_1^*$

green demand curve

II.B. Quantitative measurement of welfare losses

dp^* ? $\quad p^* \to p^* + dp^*$
$t = 0 \to t > 0$

definition of p^*:

$$y(p^*) = x(p^* + t)$$

$$\Leftrightarrow \underbrace{x(p^* + t) - y(p^*)}_{\text{excess demand function (evaluated at } p^*)} = 0$$

$dt \simeq 0 \qquad t = 0 + dt \qquad t = dt > 0$

|| $d\left[x(p^*+t)\right] = x'(p^*+t)\big|_{t=0}\, dt$

(Taylor formula) $= x'(p^*)\, t < 0$ ⇐

$$d\left[\underbrace{x(p^*+t) - y(p^*)}\right]$$

$$= x'(p^*)\, dp^* - y'(p^*)\, dp^*$$

$$= \left[x'(p^*) - y'(p^*)\right] dp^*$$ → unknown

Full effect:

$$\left[x'(p^*) - y'(p^*)\right] dp^* = -x'(p^*)\, t$$ ↑

$$\Leftrightarrow \quad dp^* = -\frac{x'}{x'-y'}\, t \quad < 0$$

dp^* ← producer price ($p+t=q$) consumers

t ← tax

(−) (+) (+)

$t > 0$

dp^* = change in the producer price when the policy maker introduces tax $t>0$

$$dq^* = dp^* + t = -\frac{x'}{x'-y'}\, t + t$$

$$= \frac{x'-y'-x'}{x'-y'}\, t$$

$$= -\frac{y'}{x'-y'}\, t \quad > 0$$

(−) (−) (+)

$$dp^* = -\frac{x'}{x'-y'}\, t = -\frac{1}{1 - \underbrace{y'/x'}}\, t$$

1) $\frac{y'}{x'} \to 0$ $\quad dp^* = -t$ "producers suffer bear the full tax"

$dq^* = dp^* + t = -t + t = 0$

$t = 0$: $p^* = q^* = 10$ €

$t = 1$ euro $\Rightarrow$ $p^* = 10 - 1 = 9$ €

$q^* = (10-1) + t = 10$ €

2) $-\frac{y'}{x'} - p + \infty$

$dp^* \simeq 0$

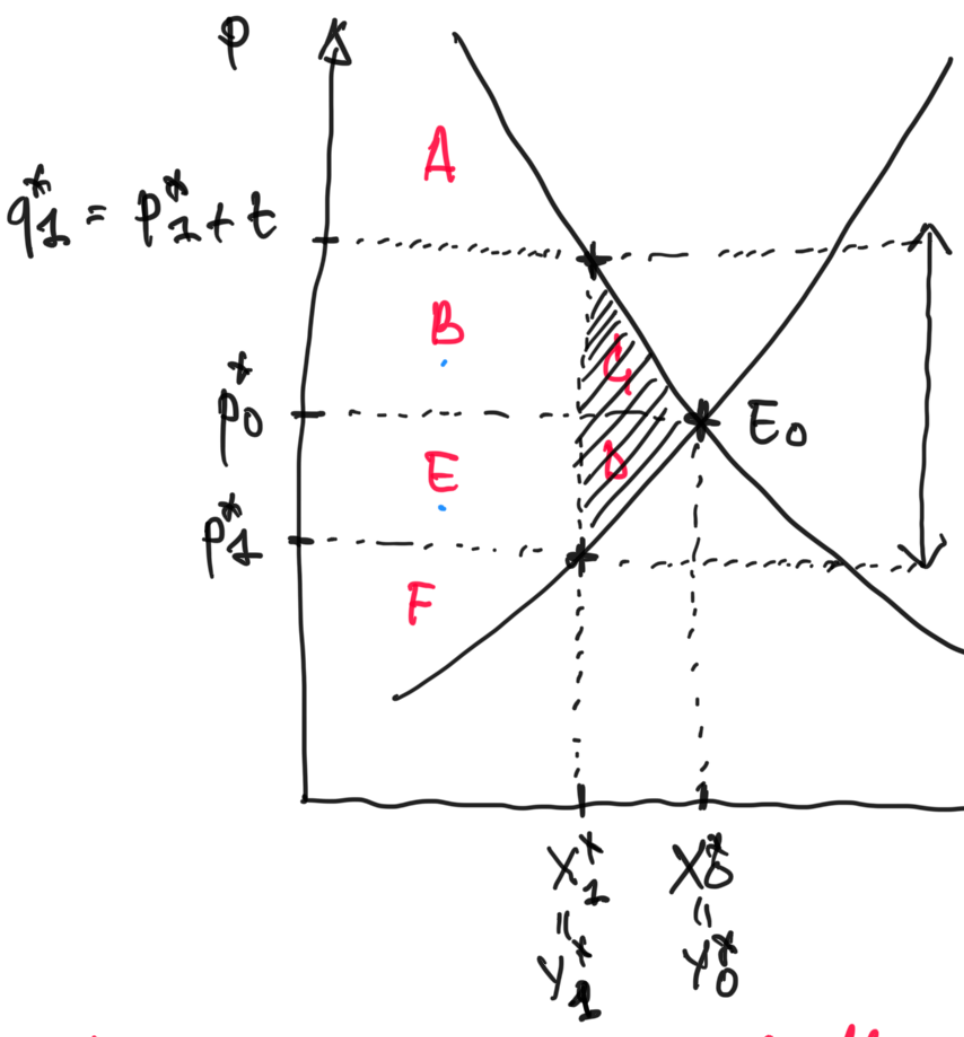
$dq^* \simeq dp^* + t \simeq 0 + t = t$

$t = 1$ euro

$p^* = 10$ €

$q^* = 10$ € $+ 1 = 11$ €

III Social welfare loss



$c'(y) = p$

Deadweight loss

$= \frac{1}{2} t (x_0^* - x_1^*)$ > 0

$= \frac{1}{2} t (-dx^*)$

$= -\frac{1}{2} t (Qt)$

$= -\frac{1}{2} t^2$ $Q > 0$ (-)

(net) consumers surplus : initially, $A + B + C$ | loss $= B + C$
finally, A

Producers (profits) : initially, $E + D + F$ | loss : $E + D$
finally, F

tax collected : $t X_1^* = t Y_1^* = B + E$

change in social welfare :

change in consumers surplus	$-(B + C)$
+ change in the profits	$-(E + D)$
+ change in the collected tax	$+(B + E)$

$0 > (a+b)-$