Chapitre 5 Biens publics

|| excluable
|| non-rival

→ bien public pur → non rival / non-excluable

G qté de biens publics
⇒ G euros (coût)

$I = 2$, $i = 1, 2$

$u_i(G) + m$ $\quad \bar{m}, \bar{M}$ $\quad u_i' > 0$, $u_i'' < 0$

Optimalité Nash Marshall

$\max_{(G)} \; u_1(G) + u_2(G) - G$

G^{opt} (si $G^{opt} > 0$) $\quad u_1'(G) + u_2'(G) - 1 = 0$

$$\boxed{u_1'(G^{opt}) + u_2'(G^{opt}) = 1}$$

Pareto $\begin{cases} \max\limits_{(G, m_1, m_2)} & u_1(G) + m_1 \quad \leftarrow \\ \text{s.c} & u_2(G) + m_2 \geq \bar{u}_2 \\ & m_1 + m_2 + G \leq \bar{M} \quad \times \end{cases}$

à l'optimum, $m_1 = \bar{M} - m_2 - G$

$$\begin{cases} \max\limits_{(G, m_2)} & u_1(G) + (\bar{M} - m_2 - G) \quad \leftarrow \\ \text{s.c} & u_2(G) + m_2 \geq \bar{u}_2 \end{cases}$$

$$m_2 = \bar{u}_2 - u_2(G)$$

$$\max_{(G)} \cancel{u_1(G)} + \bar{M} - (\bar{u}_2 - \cancel{u_2(G)}) - \cancel{G}$$

$$\Leftrightarrow \max_{(G)} u_1(G) + u_2(G) - G + (\bar{M} - \bar{u}_2)$$

$$\max_{(G)} u_1(G) + u_2(G) - G$$

(≈) $\max_{(G)}$ [ ]

Surplus marshallien

$$\boxed{u_1'(G^{opt}) + u_2'(G^{opt}) = 1}$$

règle de SAMUELSON (1952 (?))

utilité marginale sociale = coût marginal

Exemple $u_i(G) = \theta_i \ln G$ $\quad \theta_1 < \theta_2$

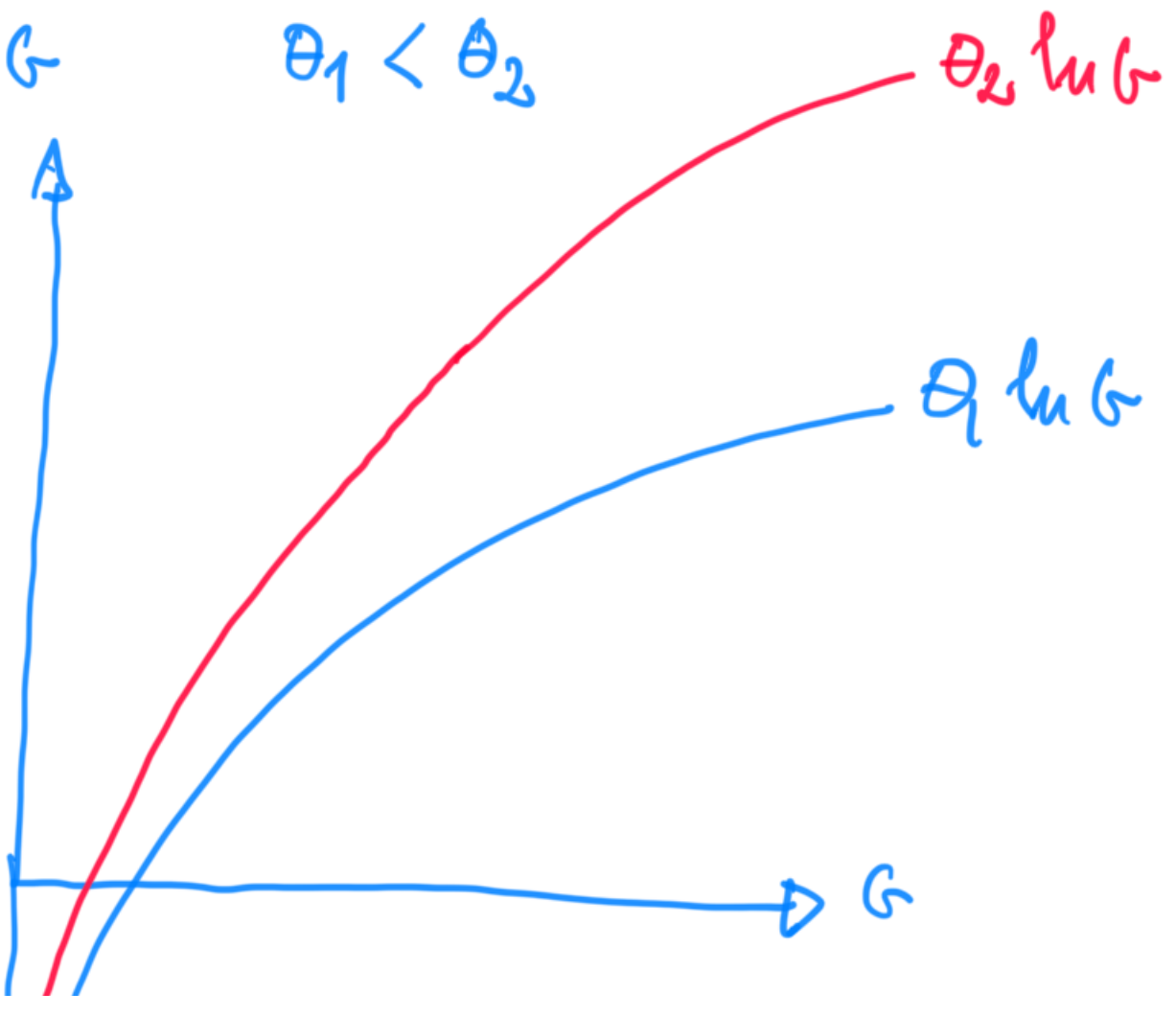
$u_i'(G) = \dfrac{\theta_i}{G}$

$u_1'(G^{opt}) + u_2'(G^{opt})$

$= \dfrac{\theta_1}{G^{opt}} + \dfrac{\theta_2}{G^{opt}} = 1$

(règle de samuelson)

$$\boxed{G^{opt} = \theta_1 + \theta_2}$$



II Contributions volontaires

g_i : contribution de l'agent i pour le bien public

$G = g_1 + g_2$

Programme de l'agent 1

$$\begin{cases} \max\limits_{(g_1)} \; u_1(\underbrace{g_1 + g_2}_{=G}) + \bar{m} - g_1 \\ g_1 \geq 0 \end{cases}$$

$g_1 = 0$ Agent 1 = "passager clandestin"

$g_1 \geq 0$, $dg_1 > 0$

$$\left[u_1'(g_1 + g_2) - 1 \right] dg_1$$

$u_1''(g_1 + g_2) < 0$

utilité Δ

$u_1(g_2) + \bar{m}$

$u_1'(g_2) + 1 < 0$

$u_1'(g_2) - 1$

$u_1(g_1 + g_2) - g_1 + \bar{m}$

0

g_1

$R_1(g_2)$

$g_1 = 0$

$u_1(g_1 + g_2) + \bar{m} - g_1$

$u_1(g_1 + g_2) + \bar{m} - g_1$

$u_1'(g_1 + g_2) - 1$

Au point $g_1 = 0$

$u_1'(0 + g_2) - 1$

$= u_1'(g_2) - 1$

Meilleur choix de contribution g_1 : $R_1(g_2)$

$$R_1(g_2) = \begin{cases} 0 \text{ si } u_1'(g_2) \leq 1 \Leftrightarrow g_2 \geq \bar{g}_2 \\ \text{(passage clandestin)} \\ u_1'(\underbrace{R_1(g_2) + g_2}) = 1 \text{ si } u_2'(g_2) \geq 1 \end{cases}$$

$\Leftrightarrow g_2 \leq \bar{g}_2$

$u_1'(g_2)$ est décroissante avec g_2

Soit $\bar{g}_2$ / $u_1'(\bar{g}_2) = 1$

$$u_1'(R_1(g_2) + g_2) = 1$$

$$\Rightarrow R_1(g_2) + g_2 = \text{cste}$$

$$\Leftrightarrow R_1(g_2) = \text{cste} - g_2$$

$$R_1(\bar{g}_2) = 0 \Leftrightarrow R_1(\bar{g}_2) = \text{cste} - \bar{g}_2 = 0$$

$$\Leftrightarrow \text{cste} = \bar{g}_2$$

$$R_1(g_2) = \bar{g}_2 - g_2$$

Soit

$$\begin{cases} 0 \text{ si } g_2 \geq \bar{g}_2 \\ \bar{g}_2 - g_2 \text{ si } g_2 \leq \bar{g}_2 \end{cases}$$

passage

$\bar{g}_2 / u_1(g_2) = 1$,

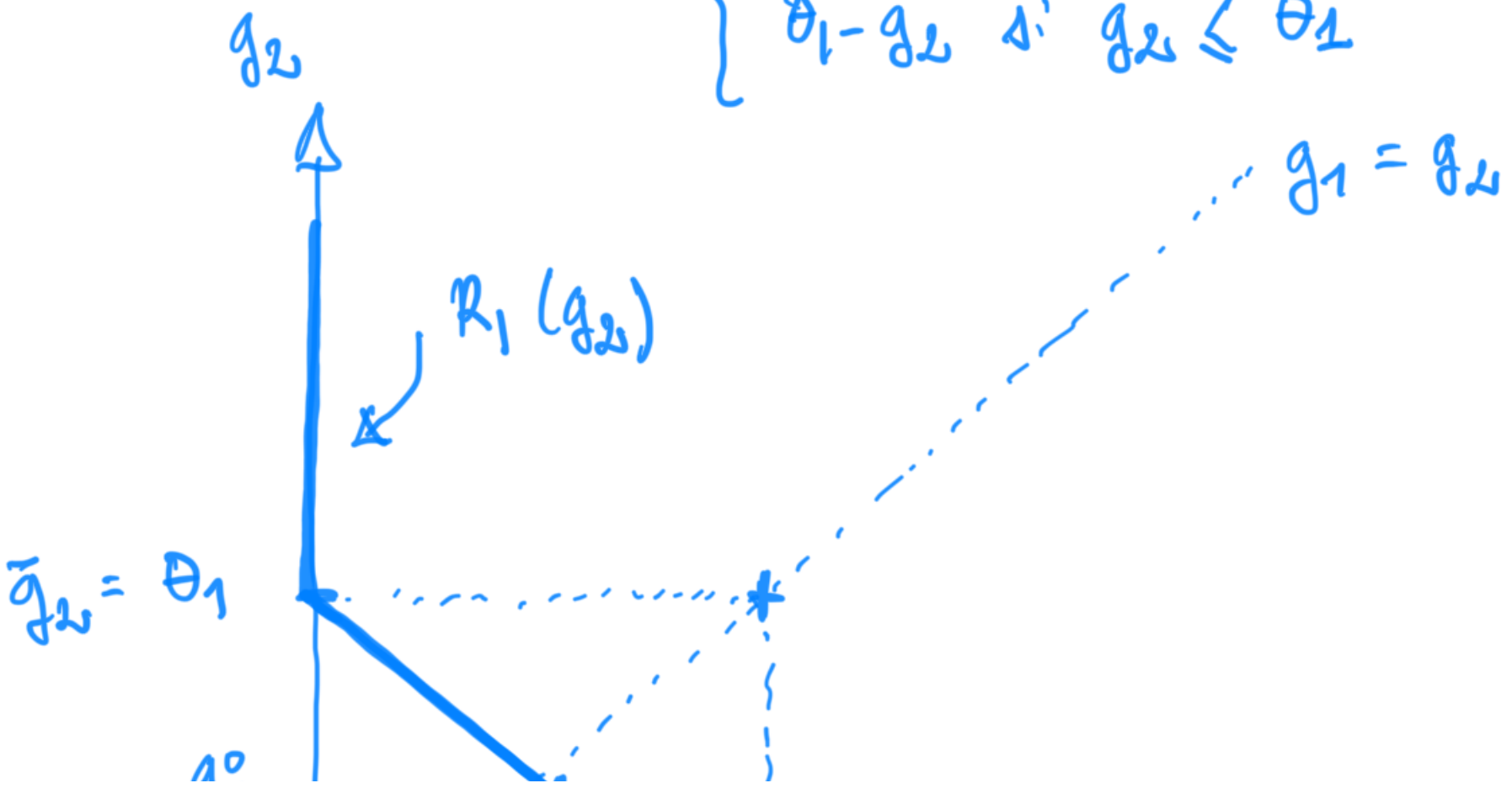
$$R_1(g_2) = \begin{cases} \\ \\ \bar{g}_2 - g_2 \text{ si } g_2 \leq \bar{g}_2 \end{cases}$$

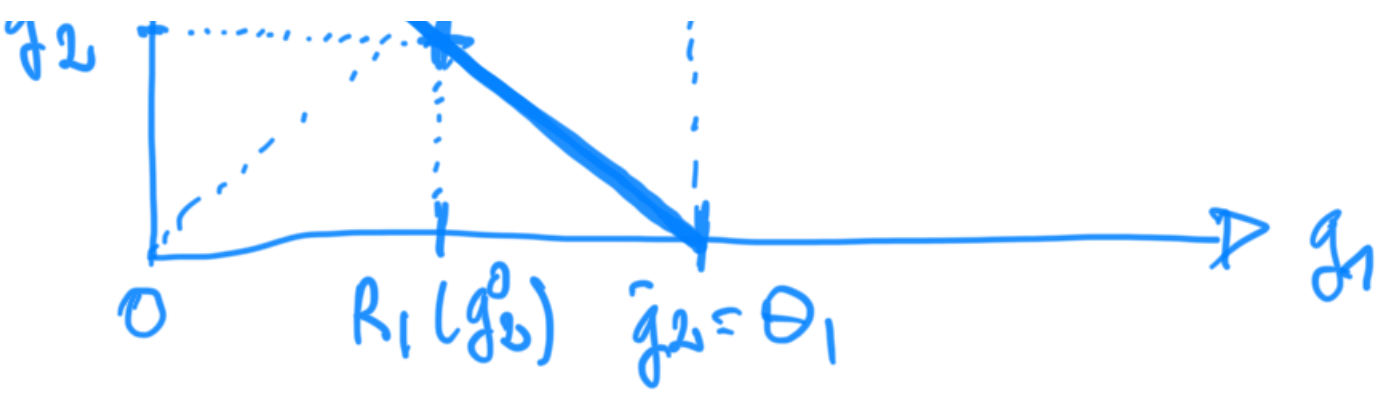
clandestin

Exemple $u_1(G) = \theta_1 \ln G$

$$u_1'(\bar{g}_2) = 1 \Leftrightarrow \frac{\theta_1}{\bar{g}_2} = 1 \Leftrightarrow \bar{g}_2 = \theta_1$$

$$R_1(g_2) = \begin{cases} 0 \text{ si } g_2 \geq \theta_1 \\ \\ \theta_1 - g_2 \text{ si } g_2 \leq \theta_1 \end{cases}$$





↳ meilleure contribution de l'agent 1 si l'agent 2 contribue g_2^0 mais.

l'agent 1 pense que

<u>comportement de l'agent 2</u> :

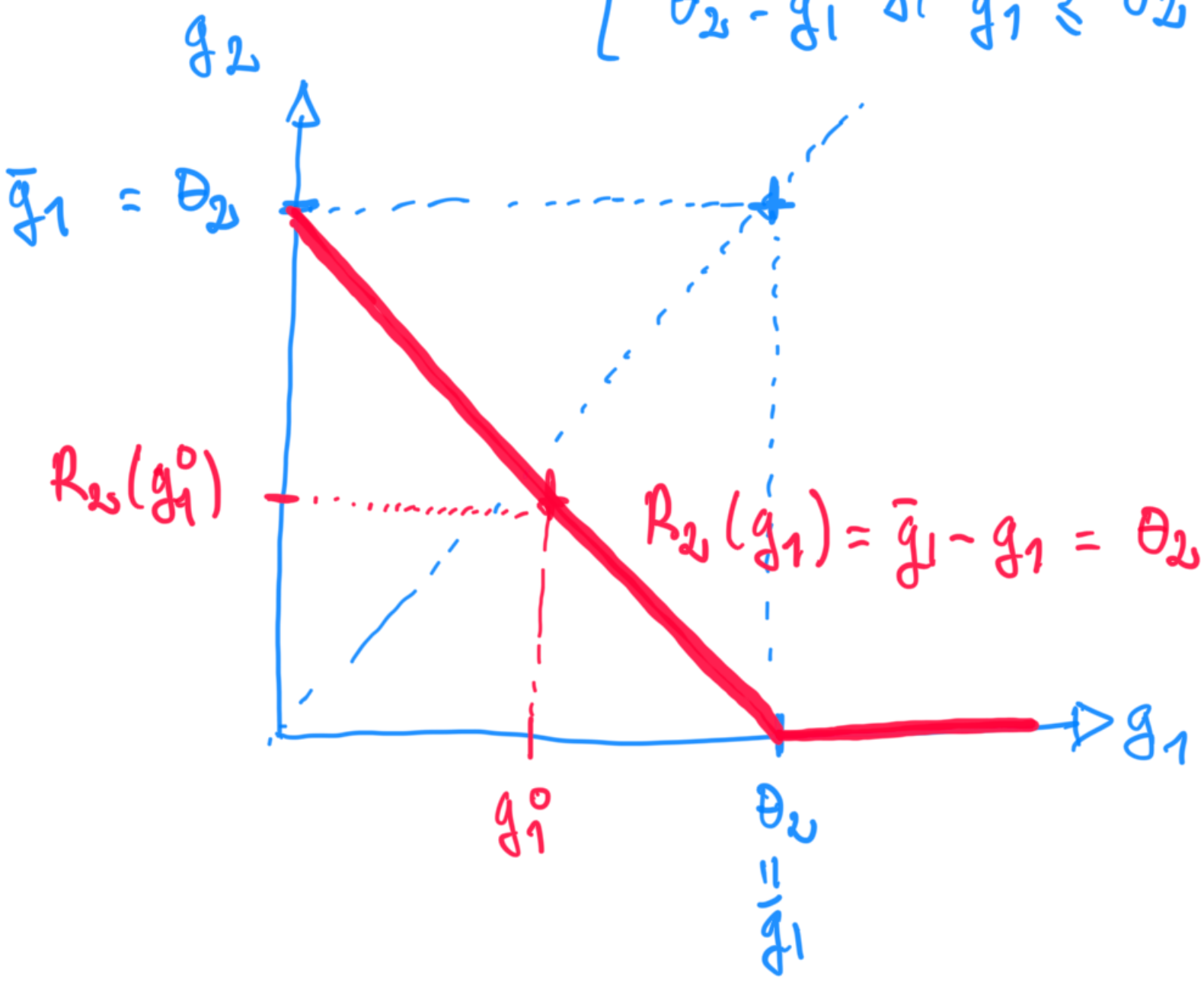
$$\max_{g_2} \; u_2(g_1 + g_2) + \bar{m} - g_2$$

$$\bar{g}_1 \;/\; u_2'(\bar{g}_1) = 1$$

$$R_2(g_1) = \begin{cases} 0 & \text{si } g_1 \geq \bar{g}_1 \\ \bar{g}_1 - g_1 & \text{si } g_1 \leq \bar{g}_1 \end{cases}$$

$u_2(\sigma) = \theta_2 \ln \sigma \Rightarrow g_1 = \theta_2$

$$R_2(g_1) = \begin{cases} 0 & \text{si } g_1 \geq \theta_2 \\ \theta_2 - g_1 & \text{si } g_1 \leq \theta_2 \end{cases}$$





Equilibre de Nash

$R_2(0) = \bar{g}_1 = \theta_2$

$g_2^2 = R_2(g_1^1)$

$\bar{g}_2 = \theta_1$

$g_2^1 = R_2(g_1^0)$

g_2^0

$R_2(g_1) = \theta_2 - g_1$

g_1

$R_1(g_2^1) = 0$ "passager clandestin"

$R_1(g_2^1)$ $R_1(g_2^0)$ θ_1 θ_2

$g_1^{1\,"}$ $g_1^{0\,"}$ $\bar{g}_2^{\,"}$ $\bar{g}_1^{\,"}$

Etape 0 : Agent 1 voit que l'agent 2 va contribuer à hauteur de g_2^0 unos. Il choisit $g_1^0 = R_1(g_2^0)$

Etape 1 : Agent 2 contribue $g_2^1 = R_2(g_1^0)$

$g_1 = 0$ $\quad g_2 = R_2(0)$

$g_1 = R_1(R_2(0)) = 0$

Equilibre de NASH

$$\begin{cases} g_1^* = 0 \\ g_2^* = \theta_2 \end{cases}$$ partage dandestin

$$G^* = \theta_2 + 0 = \theta_2 < \theta_1 + \theta_2 = G^{opt}$$

$\Rightarrow$ Eq de Nash n'est pas optimal (Pareto / Pareto)