Introduction

Course title: "Competitive Equilibrium and Market Failures"

The expression "competitive equilibrium" refers to a fundamental theoretical result obtained in the 1950s by two economists:

- Kenneth Arrow (https://fr.wikipedia.org/wiki/Kenneth_Arrow) and
- Gérard Debreu (https://fr.wikipedia.org/wiki/G%C3%A9rard_Debreu).

This fundamental result consists in the proof of an existence theorem for a general equilibrium of perfect competition.

This proof and the assumptions on which it rests make up what is called "the perfect competition model", a model at the heart of neoclassical theory (Warning! not the "pure and perfect competition model", which does not exist in the scientific literature in economics). Many other models have been built in relation to this model and its results (such as the imperfect competition models that you will study in the second semester).

Neoclassical theory is today's dominant theory in economics (dominant in terms of the number of people who subscribe to it or work within its framework, the number of articles published, of economics "Nobel prizes", etc.).

There are other theoretical currents in economics: post-Keynesian, Marxist or neo-Marxist, regulationist, conventionalist, institutionalist, Hayekian, neo-Austrian, classical, … which are on the way to disappearing (dominated). In France, most of these non-neoclassical economists are at the AFEP (Association Française d'Economie Politique).

Neoclassical theory is dominant. Including in your curriculum.

At the heart of neoclassical theory therefore lies the perfect competition model, also called the Walrasian model, the Arrow-Debreu model or general equilibrium theory (I will come back to these "or"s below to clarify things a little). This is why I will devote almost this whole semester to it.

The founding question of this model is that of the coordination of individual decisions. In microeconomics, as you saw last year, these decisions are those of consumers and producers.

In the "perfect competition" or "Arrow-Debreu" or "general equilibrium" model, the economic decisions made by consumers and producers concern the quantities of goods that they supply and demand. In this framework, to say that agents coordinate, in other words that their decisions are compatible, is to say that, for each good, the sum of the quantities supplied equals the sum of the quantities demanded. Now, when, for each good, the sum of the quantities supplied equals the sum of the quantities demanded, we say that we are at general equilibrium (hence the name general equilibrium theory, the name Walrasian model coming from the first economist to have worked out a general equilibrium model, namely Léon Walras https://fr.wikipedia.org/wiki/L%C3%A9on_Walras).

The version of this theory that lies at the heart of neoclassical theory is, however, different from the one worked out by Walras. It is the version of Kenneth Arrow and Gérard Debreu (1954, "Existence of an equilibrium for a competitive economy", *Econometrica*, vol. 22, no. 3, 265–290). Hence the name Arrow-Debreu model.

The approach of these authors may seem peculiar, even though it is typical of neoclassical theory. The question that these two theorists asked themselves – and to which they answer in the theoretical model of that article – can be stated as follows: under what conditions does a general equilibrium

exist? Since the quantities of goods supplied and demanded by agents themselves depend on the prices of the goods, the question is in fact more precisely: under what conditions do prices exist (one price per good) that equalize the (aggregate) supplies and demands of each good?[1]

But beware, the existence at stake here is not existence in reality. It is existence in the mathematical sense of the term:

> → general equilibrium requires that, for each good *i*, the quantity demanded of it $D_i(P)$, which depends on the prices of all goods ($P = (p_1, p_2, \dots, p_n)$ where p_i denotes the price of good *i*), equals the quantity supplied of it $O_i(P)$, which itself depends on P. For each good *i*, we therefore have an equation: $D_i(P) = O_i(P)$. For the whole set of the *n* goods of the economy, we thus have a system S of *n* equations whose unknown is $P = (p_1, p_2, \dots, p_n)$, namely the prices of the *n* goods.
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> → The question "under what conditions do prices exist (one price per good) that equalize the (aggregate) supplies and demands of each good?" therefore means: "under what conditions does system S have at least one solution (whose elements must a fortiori be positive since they are prices)?".

In short, when we say "under what conditions does a general equilibrium exist?" this means "which assumptions must be made in order to deduce – or prove – an existence theorem for an *n*-tuple $P = (p_1, p_2, \dots, p_n)$ of prices that equalize the aggregate supply and demand of each good?".

A set of existence conditions for an equilibrium was established in the 1950s by Arrow and Debreu (more precisely in their 1954 article cited above).

The approach of these theorists is therefore the following: they started from the result they wanted to obtain (the conclusion of the model must be that a general equilibrium exists, in other words that system S has at least one solution P, one moreover all of whose elements are strictly positive because they are prices), and they looked for the assumptions that had to be posited in order to deduce such a result from them.

Thus, for example, individuals are assumed to be able to rank all possible present and future baskets of goods according to their preferences, not because they are in reality, but because such an assumption is necessary to ensure the existence of a general equilibrium. So, to the question "why do these economists represent things this way?", the answer is often "because it was necessary to obtain the result they wanted, because it was necessary (and, moreover, sufficient) to prove the existence (in the mathematical sense of the term) of a general equilibrium". Another example: each agent is assumed to have enough to live on without carrying out any exchange (and without working in a firm). Why? For the same reason.

The idea of some of these first theorists was less to say that things happen like this in reality than to show the conditions that must be met for them to happen like this and, hence, to explain why they might happen otherwise.

Finally, every economic theory describes (explicitly or implicitly) a form of social organization. The "ideal" social organization (since it ensures the existence of a general equilibrium having, what is more, remarkable properties) of neoclassical theory is the one found in the Arrow-Debreu model,

[1] "The question addressed by Arrow and Debreu is the following: under what conditions (sufficiently general and formalized) do prices exist that simultaneously equilibrate all markets? This question is known as that of the 'existence of a general equilibrium'" (Starr, Ross M., 2008, "Arrow, Kenneth Joseph", in Steven N. Durlauf & Lawrence E. Blume ed., *The New Palgrave Dictionary of Economics*, Volume 1, Second edition, New York, Palgrave Macmillan, p. 236)

which is called perfect competition, in English *perfect competition*[2] (hence the name perfect competition model).

In this course, I will present this model to you as it is (even though I will do so in a very simplified version). Without prejudging what happens in reality: I am not going to apply this model to reality (you will do so with other instructors, you may even have already started to do so). I will just tell you what the model contains. I will therefore not talk to you about economic reality, at least not directly; I will remain on a theoretical level, i.e. that of a very specific representation of coordination.

Take, for example, the definition of "perfect competition". To know what "perfect competition" means exactly in this framework, we will come back, on the one hand, to the assumptions that make it possible, in the Arrow-Debreu model, to prove the existence of a general equilibrium and, on the other hand, to the assumptions of the so-called imperfect competition models. For example, this concept of competition does not at all presuppose atomicity. Explanation.

It is important to know these assumptions, because they are fundamental to the result they lead to, namely the possibility of a certain compatibility of individual decisions. If even one of these assumptions or conditions is not met, then the possibility of this compatibility is no longer ensured. You will see some consequences of this in the second semester.

This stance will allow me to remain almost all the time within a general equilibrium framework. But we will come back to this question, because I will sometimes have to give it up.

COURSE OUTLINE

Chapter 1 – The perfect competition model 1: the case of a pure exchange economy

Chapter 2 – The perfect competition model 2: the case of an economy with production

Chapter 3 – Pareto optimality and market failures

Some things have already been seen in L1 (first year), so we will go through them quickly.

[2]And not 'pure and perfect competition', which is not a concept of orthodox theory, but a distinctly French expression probably borrowed from François Perroux. We will therefore never speak here of 'pure and perfect' competition.

Chapter 1

The perfect competition model 1 – the case of a pure exchange economy

The general equilibrium model is fundamentally a model of exchange. This is why its content is well understood in the framework of a pure exchange economy, namely an economy without production. This is the case to which I will devote this first chapter of the course.

The decisions made by the agents of the perfect competition model concern, as recalled above, the quantities of each good that they supply and demand. While, in the general model, the agents are either consumers or producers, in a pure exchange economy, by definition, there is no producer. The only agents we will meet in this chapter are therefore consumers who exchange – or not – among themselves what they own.

I. The consumer of the perfect competition model (reminders)

In the perfect competition model (as in all of neoclassical theory), the consumer is characterized by his tastes and by what he owns. What he owns is what we call his **initial endowments**. His tastes are formalized by a **preference relation** between baskets of goods.

We will therefore begin with a few brief reminders on the notion of a basket of goods, on the consumer's preferences and on some notation.

1. Baskets of goods

A good is characterized by three parameters: its physical properties, and the date and place at which the good is available.

The quantity of good *i* is generally denoted q_i. And q_i is taken to be a non-negative real number:

- It is a non-negative number because it is a quantity;
- It is a real number for reasons of mathematical simplicity (a necessary condition for continuity, we will come back to this). The mathematical form given to theoretical concepts generally has an economic meaning. Here, saying that the quantity of a good is a real number is to assume that this good – or rather its quantity – is infinitely divisible. In reality this is sometimes almost true of the good: flour is infinitely divisible… or almost. It is more often true of its quantity, because the unit of measurement is infinitely divisible: one can very well, for example, measure the quantity of apples in kg (infinitely divisible). But it is sometimes false: 1.23 computers, cars, houses, pens, shoes, etc. do not exist. If goods were not divisible, their quantity would be denoted by a natural integer.

In short, a good *i* is a set of characteristics including the place and date of availability, and its quantity is assumed to be infinitely divisible, so that its quantity, q_i, is a non-negative real number.

Basket of goods – In an economy with *n* goods, a basket of goods Q is the *n*-tuple or vector:

$$Q = (q_1, \dots, q_n),$$

where q_i denotes the quantity of good *i*, $i = 1, \dots, n$.

As you did in L1, we will most often work with baskets containing two types of goods, $Q = (q_1, q_2)$, in order to simplify the presentation and to produce graphical representations. But this does not change most of the economic reasoning.

As said above, the consumer is characterized by his initial endowment and his preferences.

The **initial endowment** is the set of resources that the consumer has at his disposal for his consumption and for making exchanges. Beware, there is no money. The initial endowment therefore takes the form of a basket of goods Q^0: in the two-good world, good (1) and good (2), this basket may be (3 , 6) for example, in which case the agent has a quantity equal to 3 of good (1) and a quantity 6 of good (2).

Remark: the model does not ask the question of – and says nothing about – the origin of these initial endowments, nor the reason for their amount (we will come back to this point because it is not entirely accurate, we will see later why). Be that as it may, the initial distribution of resources is given.

2. The consumer's preferences

The preference relation

The consumer's tastes are described by a preference relation, denoted $\succsim$, which means "preferred or indifferent to". Thus we write:

$$Q \succsim Q',$$

to say that the consumer either prefers Q to Q' in the strict sense ($Q \succ Q'$) or is indifferent between Q and Q' ($Q \sim Q'$). Finally, $Q \succsim Q'$ means that the consumer prefers Q to Q' in the weak sense.

This preference relation is defined on the set of baskets of goods. And it has three mathematical properties:

- It is **complete**: for any baskets Q and Q' considered, the consumer says either that he prefers Q to Q', or that he prefers Q' to Q, or that he is indifferent between consuming one or the other. In other words, for the preference relation to be complete, the consumer must not say, when faced with any two baskets: I don't know how to rank them.
- It is **reflexive**: for any basket Q, we have: $Q \succsim Q$.
- It is **transitive**: for any baskets Q, Q' and Q'', we have:

  $$Q \succsim Q' \text{ and } Q' \succsim Q'' \Rightarrow Q \succsim Q''$$

  (if the consumer prefers Q to Q' and Q' to Q'', then he prefers Q to Q'': consistency of preferences).

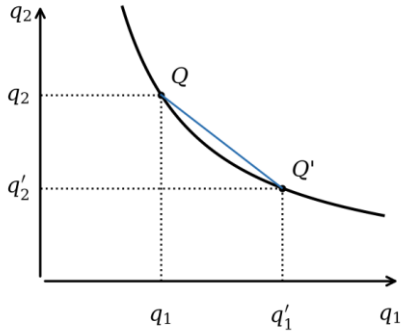
A preference relation having these various properties is what mathematicians call a **complete preorder**. The difference between a preorder and an order is that in a preorder, two different alternatives Q and Q' may be considered equivalent (or "indifferent"), whereas in an order they are equivalent only if they are identical.

Two baskets considered equivalent in this way are on the same indifference curve.

Indifference curves

In a two-good economy, an **indifference curve** is a curve linking all the baskets that give the consumer the same satisfaction, and between which he is consequently indifferent.

Indifference curves are generally assumed to be continuous, strictly decreasing, convex and asymptotic to the axes. Graphically, these properties translate into curves that have the shape below.



These mathematical properties translate the following economic assumptions: goods are infinitely divisible and substitutable, the consumer always prefers more to less (non-satiation hypothesis), all goods are desirable and "the consumer prefers mixtures" of baskets of goods (convexity of preferences hypothesis). If you have forgotten this, you absolutely must read the appendix of this course (pp. 10-12).

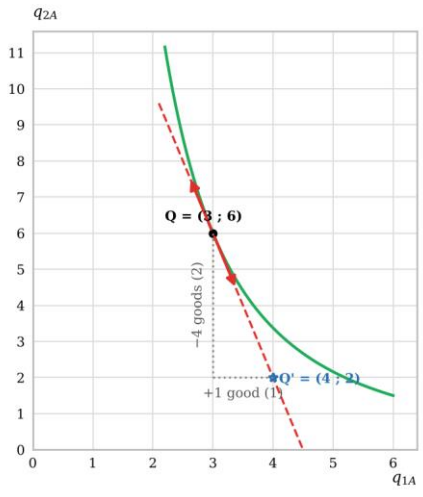
Warning to carefully distinguish between *a mathematical property*, *a graphical representation* and *an economic assumption*.

The set of indifference curves relating to a preference relation is called an **indifference map**.

The marginal rate of substitution

The **marginal rate of substitution of good (2) for good (1)** *[or between good (1) and good (2), since either of the two can be substituted for the other]*, denoted $MRS_{2/1}(Q)$, is "approximately" the quantity of good (2) that the consumer must give up in exchange for one unit of good (1) in order to keep the same satisfaction (*i.e.* to stay on the same indifference curve).

Graphically, the marginal rate of substitution (hereafter MRS) corresponds to the absolute value of the slope of the tangent at Q.



On the graph at left, the slope of the tangent to the indifference curve at Q is equal to – 4 (the absolute value of the slope of a triangle = height/base). Hence: $MRS_{2/1}(Q) = 4$: when he owns the basket $Q = (3 , 6)$, the agent is willing to give up 4 goods (2) to obtain one more good (1).

In fact, this is not exactly right. Hence the "approximately" in the definition. Indeed, if he gives up 4 goods (2) for one good (1), the agent ends up with the basket $Q' = (4 , 2)$, located below the indifference curve passing through Q: he therefore strictly prefers Q to Q', so that he will not be willing to make this exchange. This is obviously because, if we speak of *marginal* rate of substitution, it is because its result is valid only for a *very small* quantity of good.

Hence the definition from the textbook by M. Pucci & J. Valentin (2010):

> The marginal rate of substitution of good (2) for good (1) is the "quantity of good (2) that the consumer must give up per unit of good (1) in order to acquire a "marginal" quantity of good (1) while keeping the same level of satisfaction."

End of the week 1 lecture

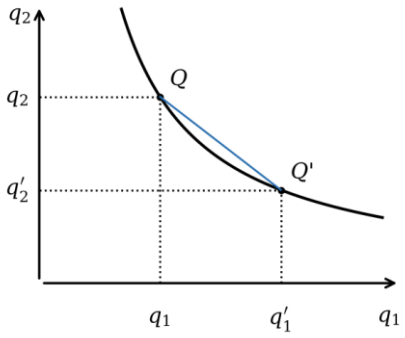
- Read the appendix if you have forgotten the L1 course

Appendix – Reminders on indifference curves

Consider an economy with two goods, (1) and (2). And consider an individual A able to rank, according to his preferences, all possible baskets of goods (q_1, q_2).

1. Usual shape of indifference curves

The indifference curves of an individual A are generally assumed to be continuous, decreasing, strictly convex and asymptotic to the axes. Indifference curves therefore have the following shape:



Reminder: a curve is strictly convex when, for any points Q and Q' of the curve, the segment [QQ'] lies above the curve.

Which economic assumption(s) do these mathematical properties translate?

Continuity mainly translates the following "economic" assumptions: goods are infinitely divisible and A is able to rank, according to his preferences, all possible baskets of goods. These two assumptions more precisely have the consequence that an indifference curve passes through each of the points of the first quadrant of the plane, which represents the set of possible baskets.

The **decrease** translates the assumption known as "non-satiation of wants", stipulating that the individual always prefers more to less.

Thus, for example, between the two baskets Q_1 = (1 , 1) and Q_2 = (2 , 2), the individual prefers Q_2, which contains both more of good (1) and more of good (2) than Q_1. Likewise, to basket Q_1, the individual prefers Q_3 = (1 , 2), which contains as much of good (1) but more of good (2) than Q_1, or Q_4 = (2 , 1), which contains as much of good (2) but more of good (1) than Q_1.

On a graph, we can thus draw 4 zones (I, II, III and IV) around any basket Q:

- **Zone I: north-east (NE)** of Q, including the baskets located on the vertical half-line above Q and on the horizontal half-line to the right of Q. All the baskets located to the NE of Q contain more of good (1) and of good (2) than Q. All the baskets located on the vertical half-line above Q contain as much of good (1) as Q, but more of good (2). All the baskets located on the horizontal half-line to the right of Q contain as much of good (2) as Q, but more of good (1). All the baskets in this zone are therefore preferred by A to basket Q. It follows that the indifference curve of A passing through Q cannot also pass through a basket in this zone (the non-satiation hypothesis forbids it).
- **Zone II: south-west (SW)** of Q, including the baskets located on the vertical half-line below Q and on the horizontal half-line to the left of Q. All the baskets located to the SW of Q contain less of good (1) and of good (2) than Q. All the baskets located on the vertical half-line below Q contain as much of good (1) as Q, but less of good (2). All the baskets located on the vertical half-line to the left of Q contain as much of good (2) as Q, but less of good (1). To all the baskets in this zone, agent A therefore prefers basket Q. It follows that the indifference curves passing through Q cannot also pass through a basket in this zone (the non-satiation hypothesis forbids it).

- That leaves **zones III** (north-west (NW) of Q) and **IV** (south-east (SE) of Q): the baskets of zone III contain more of good (2), but less of good (1) than Q. The non-satiation hypothesis therefore does not forbid the indifference curve passing through Q from also passing through this zone. Same for zone IV, whose baskets contain more of good (1), but less of good (2).

If we apply this reasoning to all the points of the curve, then the latter can only be strictly decreasing.

The strict decrease of indifference curves translates another assumption: the **substitutability** of goods. The strict decrease of indifference curves means, indeed, that if the individual is indifferent between two distinct baskets Q and Q', then Q contains strictly more of good (1) and strictly less of good (2) than Q', or vice versa. Some good (2) can thus be substituted for some good (1) (or vice versa) without the individual's satisfaction being changed.

From these first two properties, two consequences can be deduced:

- indifference curves cut the first quadrant of the plane into two zones: a zone, to the right of or above the curve, containing only baskets preferred to those of the curve, and a zone, to the left of or below the curve, containing only baskets to which the consumer prefers the baskets of the indifference curve.
- Two indifference curves cannot cross: because, if they cross, then the non-satiation hypothesis is not respected, unless choices are not transitive.

Indifference curves are **asymptotic to the axes**: that they get closer and closer to the axes translates the fact that they are decreasing and that all the points of the first quadrant lie on an indifference curve. But this decrease does not ensure that indifference curves do not cross the axes. The fact that indifference curves do not cross the axes translates another assumption on preferences: the assumption of **desirability of goods**. According to this assumption, if we have two baskets, one of which, Q, does not contain a good while the other, Q', does, then the second is preferred to the first, whatever the (non-zero) quantity of the other good that each of the baskets contains.

Convexity of indifference curves. The indifference curve passing through two baskets Q and Q' is convex if and only if the segment [QQ'] lies above the curve. In other words, any basket located on the segment [QQ'] is preferred to the baskets of the curve passing through Q and Q'.

Now the baskets of the segment [QQ'] are of the form: $\lambda Q + (1 - \lambda)Q'$, with $\lambda \in]0 , 1[$.

If the baskets of the form $\lambda Q + (1 - \lambda)Q'$ (with $\lambda \in]0 , 1[$) are systematically and strictly preferred to Q and to Q', we say that "the consumer prefers mixtures" (implicitly, mixtures of baskets) — indeed, a **mixture** of baskets Q and Q', where Q and Q' are any two baskets, is any basket of the form $\lambda Q + (1 - \lambda)Q'$, where λ is a real number between 0 and 1 (which implies that $1 - \lambda$ is also between 0 and 1).

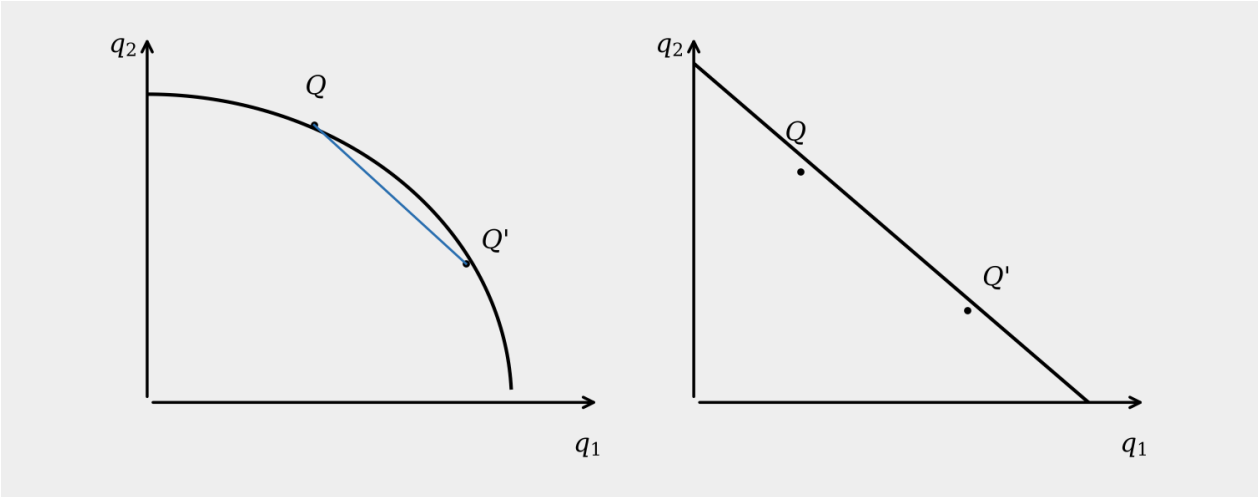
We also speak of **convexity of preferences** to designate this assumption on individuals' tastes.

2. Unusual shapes of indifference curves

a. Suppose that the indifference curves of an individual B are continuous, decreasing and concave. Which economic assumptions do these mathematical properties translate?

Same as above for "continuous" and "decreasing".

Concavity (in the weak sense) refers, for its part, to the fact that individual B prefers (in the weak sense) baskets Q and Q' to any "mixture" of Q and of Q'.

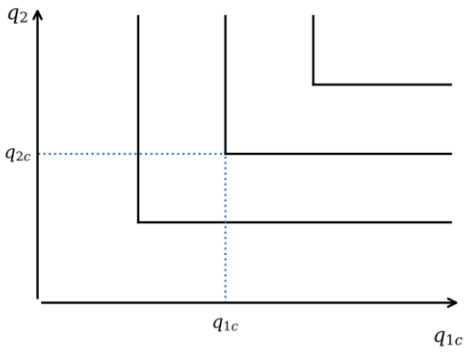


b. Suppose that the indifference curves of an individual C are continuous and kinked. Which economic assumption(s) do these mathematical properties translate?

For individual C, the **goods** are not substitutable, but **complementary**.

Let (q_{1c} , q_{2c}) denote the basket located at the kink. Starting from this basket,

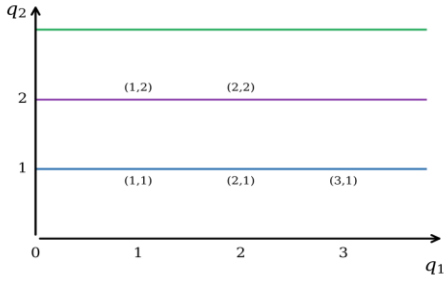
- decreasing the quantity of one of the two goods decreases the individual's satisfaction even if the quantity of the other good increases;
- increasing the quantity of one good without increasing that of the other does not increase the individual's satisfaction (since we then stay on the same indifference curve).



c. Suppose that the indifference curves of an individual D are **horizontal**. Which economic assumption does this mathematical property translate?

Individual D **likes only good (2)** and is indifferent to the quantity of good (1) that he has. Indeed, all the baskets containing a quantity $\bar{q}_2$ of good (2) give him the same satisfaction, whatever the quantity of good (1) they contain. For example, the baskets (0 , 1), (1 , 1), (2 , 1), (3 , 1) are on the same indifference curve (blue at right), just as the baskets (0 , 2), (1 , 2), (2 , 2) are on the same indifference curve (purple below), etc.

Utility function representing D's preferences: $u(q_1 , q_2) = \alpha q_2$.



d. What if the indifference curves of an individual E are **vertical**?

Individual E **likes only good (1)** and is indifferent to the quantity of good (2) that he has. Indeed, all the baskets containing a quantity $\bar{q}_1$ of good (1) give him the same satisfaction, whatever the quantity of good (2) they contain.