II Externalities

Unequal / unbalanced

Agent 1 pollutes agent 2

$u(x_1) + w_1$

$u(x_2) + w_2 - \theta_2 E \qquad E = x_1$

Marshallian optimum $\quad E = x_1$; constant marginal cost

$$u(x_1) + u(x_2) - \theta_2 x_1 - c(x_1 + x_2)$$

$$\begin{cases} u'(x_1^{opt}) = c + \theta_2 \\ u'(x_2^{opt}) = c \end{cases}$$

$$u'(x_1^{opt}) > u'(x_2^{opt}) \Leftrightarrow x_1^{opt} < x_2^{opt}$$

Equilibrium

$p \qquad p^* = c$

$$\begin{cases} \max\limits_{(x_1, w_1)} \; u(x_1) + w_1 \\ \text{s.t.} \quad p x_1 + w_1 \leq \bar{w} \end{cases}$$

$w_1 = \bar{w} - p x_1 = \bar{w} - c x_1$ ($p^* = c$)

$$\max_{(x_1)} \; u(x_1) + [\bar{w} - c x_1]$$

$$u'(x_1^*) = c$$

$$\max_{(x_2)} \; u(x_2) + \bar{w} - c x_2 - \theta_2 E$$

$$\Leftrightarrow \max_{(x_2)} \; u(x_2) - c x_2$$

$$\Rightarrow u'(x_2^*) = c$$

$$u'(x_1^*) = u'(x_2^*) = c \Leftrightarrow x_1^* = x_2^*$$

$$\begin{cases} u'(x_1^{opt}) = c + \theta_2 > c = u'(x_1^*) \Rightarrow x_1^{opt} < x_1^* \\ u'(x_2^{opt}) = c = u'(x_2^*) \Rightarrow x_2^* = x_2^{opt} \end{cases}$$

Pigovian taxation t_i

$$\max_{(x_1)} \; u(x_1) - (c + t_1)\, x_1 \qquad \text{tax paid} = t x_1$$

$$u'(x_1^*) = c + t_1$$
$$u'(x_1^{opt}) = c + \theta_2 \quad \Big| \quad x_1^* = x_1^{opt} \Leftrightarrow t_1 = \theta_2 > 0$$

Pigovian

$$\max_{(x_2)} \; u(x_2) - (c + t_2)\, x_2$$

$$u'(x_2^*) = c + t_2$$
$$u'(x_2^{opt}) = c \quad \Big| \quad x_2^* = x_2^{opt} \Leftrightarrow \underline{t_2 = 0}$$

Pigovian

III Bargaining

legal right

(*) right to a clean environment

contract proposed by agent 2 to agent 1 is :

$\begin{cases} x_1 \\ T \end{cases}$ Monetary transfer from agent 1 to agent 2

Agent 1

$\begin{cases} \text{if accepts} : u(x_1) + \bar{m} - c x_1 - T \\ \text{if refuses} : u(0) + \bar{m} \end{cases}$

Acceptation : $u(x_1) + \bar{m} - c x_1 - T \geq u(0) + \bar{m}$

$$\Leftrightarrow \boxed{u(x_1) - c x_1 - T \geq u(0)} \leftarrow$$

Agent 2

$$\max_{(x_2, (x_1, T))} \; u(x_2) + \bar{m} - c x_2 - \underbrace{\theta_2 x_1}_{\text{damage caused by agent 1}} + T$$

$$T = u(x_1) - c x_1 - u(0)$$

$$\max_{(x_1, x_2)} \; u(x_2) + \bar{m} - c x_2 - \theta_2 x_1 + \left[u(x_1) - c x_1 - u(0) \right]$$

$$u'(x_1) = c + \theta_2 = u'(x_1^{opt}) \Leftrightarrow x_1 = x_1^{opt}$$
$$u'(x_2) = c = u'(x_2^{opt}) \Leftrightarrow x_2 = x_2^{opt}$$

(*) Right to a dirty environment

Agent 2 offers a contract to agent 1 :

$\begin{cases} x_1 & \text{qty consumed by agent 1} \\ T & \text{paid by agent 2 to agent 1} \end{cases}$

If agent 1 accepts : $u(x_1) + \bar{m} - cx_1 + T$

refuses : $u(x^*) + \bar{m} - cx^*$

Accept $\Leftrightarrow u(x_1) + \bar{m} - cx_1 + T \geq u(x^*) + \bar{m} - cx^*$

$\Leftrightarrow \boxed{u(x_1) - cx_1 + T \geq u(x^*) - cx^*}$ ←

Agent 2 $\max_{(x_2, (x_1, T))} \; u(x_2) + \bar{m} - cx_2 - \underbrace{\theta_2 x_1}_{\text{damage}} - T$ ←

$T = u(x^*) - cx^* - u(x_1) + cx_1$

$\max_{(x_2, x_1)} \; u(x_2) + \bar{m} - cx_2 - \theta_2 x_1 - [u(x^*) - cx^* - u(x_1) + cx_1]$

$u'(x_1^*) = c + \theta_2 \Leftrightarrow x_1^* = x_1^{opt}$

$u'(x_2^*) = c \Leftrightarrow x_2^* = x_2^{opt}$

⇒ Coase theorem

Chapter 5 Public goods

rival : If someone consumes the goods, then no other agent can consume it.

exclusion : It is possible to exclude someone from consumption.

⇒ "private goods"

⇒ pure public goods (non-rival / non-excludable)

$I = 2, \; i = 1, 2$

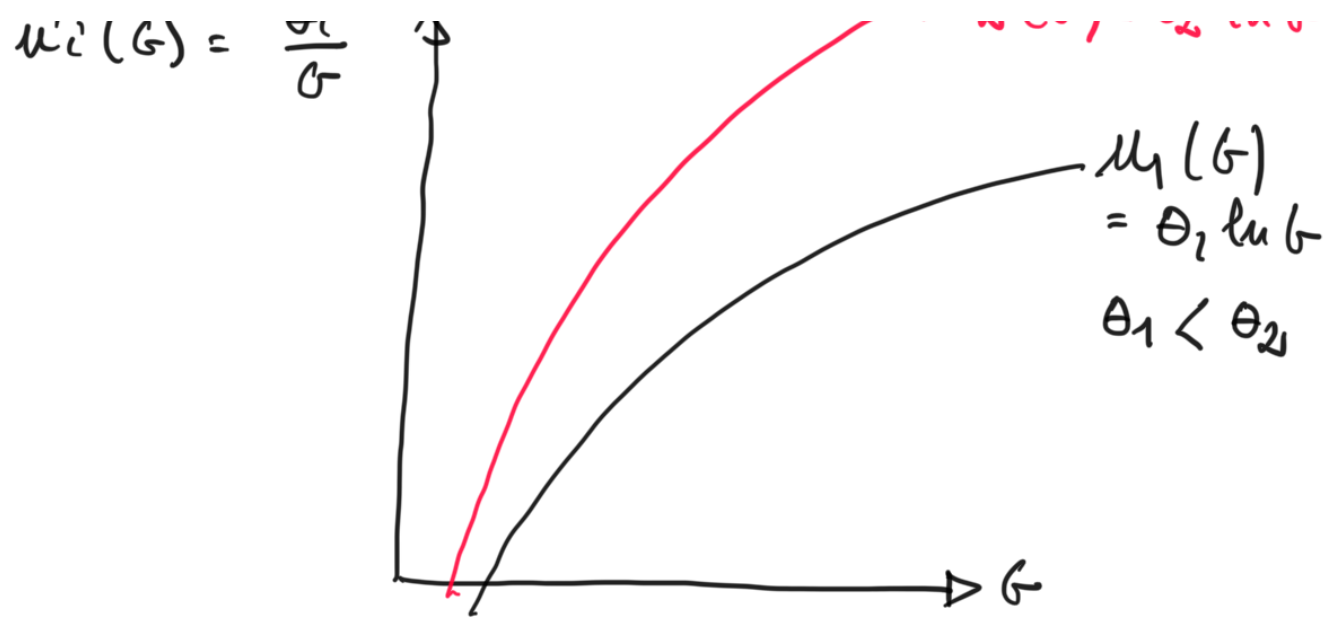
$u_i(G) + m \qquad u_i'(G) > 0, \; u_i''(G) < 0$

↑ ↳ qty of pure public good

$u_i(G) = \theta_i \ln G \;/\; \theta_i \sqrt{G}$

θ_i high ⇒ high valuation for the public good

$u_2(G) = \theta_2 \ln G$



I Optimal provision of public good G^{opt}

A. Marshallian view

Production : 1 unit of public good = 1 euro.

$$\max_{(G)} \; u_1(G) + u_2(G) - G$$

$$\boxed{u_1'(G^{opt}) + u_2'(G^{opt}) = 1}$$

B. Lindian view

$$\begin{cases} \max\limits_{(u_1, u_2, G)} & u_1(G) + u_1 \\ \text{s.t.} & u_2(G) + u_2 \geq \bar{u}_2 \\ & u_1 + u_2 + G \leq \bar{M} \end{cases}$$

$$u_1 = \bar{M} - u_2 - G$$

$$\begin{cases} \max\limits_{(u_2, G)} & u_1(G) + [\bar{M} - u_2 - G] \\ \text{s.t.} & u_2(G) + u_2 \geq \bar{u}_2 \end{cases}$$

$$u_2 = \bar{u}_2 - u_2(G)$$

$$\max_{(G)} \; u_1(G) + \bar{M} - [\bar{u}_2 - u_2(G)] - G$$

$$u_1'(G) + u_2'(G) - 1 = 0$$

$$\left| u_1'(G^{opt}) + u_2'(G^{opt}) = 1 \right|$$

Marginal
Social utility

marginal cost

⇒ "Samuelson rule"