SESSION TOPIC: INVESTMENTS—THEORETICAL ISSUES

Session Chairperson: Richard Brealey*

INFORMATIONAL SYMMETRIES, FINANCIAL STRUCTURE, AND FINANCIAL INTERMEDIATION

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Introduction and Summary

Numerous markets are characterized by informational differences between buyers and sellers. In financial markets, informational asymmetries are particularly pronounced. Borrowers typically know their collateral, industriousness, and moral rectitude better than do lenders; entrepreneurs possess “inside” information about their own projects for which they seek financing.

Lenders would benefit from knowing the true characteristics of borrowers. But moral hazard hampers the direct transfer of information between market participants. Borrowers cannot be expected to be entirely straightforward about their characteristics, nor entrepreneurs about their projects, since there may be substantial rewards for exaggerating positive qualities. And verification of true characteristics by outside parties may be costly or impossible.

Without information transfer, markets may perform poorly. Consider the financing of projects whose quality is highly variable. While entrepreneurs know the quality of their own projects, lenders cannot distinguish among them. Market value, therefore, must reflect average project quality. If the market were to place an average value greater than average cost on projects, the potential supply of low quality projects may be very large, since entrepreneurs could foist these upon an uninformed market (retaining little or no equity) and make a sure profit. But this argues that the average quality is likely to be low, with the consequence that even projects which are known (by the entrepreneur) to merit financing cannot be undertaken because of the high cost of capital resulting from low average project quality. Thus, where substantial information asymmetries exist and where the supply of poor projects is large relative to the supply of good projects, venture capital markets may fail to exist.

For projects of good quality to be financed, information transfer must occur. We have argued that moral hazard prevents direct information transfer. Nonetheless, information on project quality may be transferred if the actions of entrepreneurs (“which speak louder than words”) can be observed. One such action, observable because of disclosure rules, is the willingness of the person(s) with inside information to invest in the project or firm. This willingness to invest may serve as a signal to the lending market of the true quality of the project; lenders will place a value on the project that reflects the information transferred by the signal.

As shown by the seminal work of Akerlof [1970] and Spence [1973], and by the

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subsequent contributions of Rothschild and Stiglitz [1975] and Riley [1975, 1976],
equilibrium in markets with asymmetric information and signalling may have quite
different properties from equilibrium either with no information transfer, or with
direct and costless information transfer. Signalling equilibria may not exist, may
not be sustainable, and may not be economically efficient.

In subsequent sections, we develop a simple model of capital structure and
financial equilibrium in which entrepreneurs seek financing of projects whose true
qualities are known only to them. We show that the entrepreneur's willingness to
invest in his own project can serve as a signal of project quality. The resulting
equilibrium differs importantly from models which ignore informational
asymmetries. The value of the firm increases with the share of the firm held by the
entrepreneur. In contrast with Modigliani and Miller [1958], the financial structure
of the firm typically will be related to project or firm value even when there are no
taxes.¹ And firms with riskier returns will have lower debt levels even when there
are no bankruptcy costs. Signaling incurs welfare costs by inducing entrepreneurs
to take larger equity positions in their own firms than they would if information
could be directly transferred; we show, however, that the set of investment projects
which are undertaken will coincide with the set which would be undertaken if
direct information transfer were possible. Finally, we suggest that financial inter-
mediation, which is difficult to explain in traditional models of financial equi-
librium, can be viewed as a natural response to asymmetric information.

FINANCIAL STRUCTURE AND INSIDE INFORMATION: A SIGNALING MODEL

Consider an investment project which involves a capital outlay $K$ and a future
return $\mu + \bar{x}$, where $\mu$ is the expected end-of-period value of the project and $\bar{x}$ is a
random variable with zero mean and variance $\sigma^2$. We shall consider an en-
trepreneur who wants to undertake this investment project and plans to hold a
fraction $\alpha$ of the firm's equity, raising the remainder of the equity from other
lenders. Throughout our analysis, the firm and the entrepreneur (on personal
account) are both assumed to be able to issue debt at the riskless rate.²

The entrepreneur has information that leads him to assign a specific value to $\mu$,
but he has no credible way to convey this information directly to other potential
shareholders, who have a subjective probability distribution for $\mu$. However, other
potential shareholders will respond to a signal by the entrepreneur regarding his
evaluation of $\mu$ if they know that it is in the self-interest of the entrepreneur to send
true signals. The signal which we shall examine is $\alpha$, the fraction of the equity in
the project which is retained by the entrepreneur. This will be taken by other
lenders as a (noiseless) signal of the true $\mu$. That is, the market perceives $\mu$ to be a
function of $\alpha$.

Assuming that capital markets are competitive and that there is no uncertainty

¹ A recent study by Jensen and Meckling [1975] emphasizing management costs without considera-
tion of informational asymmetries, and a study by Ross [1976] emphasizing managerial incentives in the
presence of informational asymmetries, provide examples of alternative approaches to some of the
financial structure questions addressed in this paper.

² This assumption is not unreasonable if entrepreneurs have substantial initial wealth and project
returns are bounded below.
about the project's mean, given signal $\alpha$, we can express the total market value of the project, $V$, as

$$V(\alpha) = \frac{1}{(1 + r)} \left[ \mu(\alpha) - \lambda \right],$$  \hspace{1cm} (1)

where $r$ = the riskless interest rate;
$\mu(\alpha)$ = the market valuation schedule, expressing the market's perception of the true expected return as a function of $\alpha$, the fraction of equity retained by the entrepreneur.
$\lambda$ = the market's adjustment for the risk of the project with returns $\bar{x}$ about the mean.$^3$

We shall assume that $\mu(\alpha)$ is a differentiable function.$^4$

In addition to the possibility of investing in his own project, the entrepreneur can invest in the market portfolio. Define

- $\tilde{M}$ = the random (gross) return of the market portfolio;
- $V_M$ = the value of the market portfolio;
- $\beta$ = the fraction of the market portfolio held by the entrepreneur.

We shall make the "perfect competition" assumption that the project is small relative to the market as a whole; the entrepreneur perceives his decisions with respect to the project to have a negligible effect on the returns and value of his share of the market portfolio.

The entrepreneur is presumed to maximize his expected utility of wealth with respect to (a) the financial structure of the project or firm; (b) his holding of equity in the project or firm; and (c) his holding of the market portfolio and the riskless asset. His choices must satisfy his budget constraint

$$W_0 + D + (1 - \alpha) \left[ V(\alpha) - D \right] - K - \beta V_M - Y = 0, \hspace{1cm} (2)$$

if he undertakes the project.$^5$ In this case, end of period wealth $\tilde{W}_1$ is determined

3. Equation (1) can be shown by arbitrage, since in competitive markets an asset with return $\mu + \bar{x}$ will have the value of an asset with sure return $\mu$ plus the value of an asset with return $\bar{x}$. In the case of the capital asset pricing model, $\lambda = \lambda^* \text{Cov}(\bar{x},\tilde{M})$, where $\lambda^*$ is the "market price of risk" and $\tilde{M}$ is the return of the market portfolio.

4. At considerable complication, our theorems can be shown to hold when $\mu(\alpha)$ is differentiable almost everywhere. Thus continuity of the optimal schedule is not an essential assumption.

5. Some attention to the entrepreneur's budget constraint is required. He is assumed to have initial wealth $W_0$. The undertaking of his investment project requires an investment of $K$, and generates returns $\bar{x} + \mu$. The entrepreneur can sell claims to this return. Let $D$ represent the amount of priority claims sold (debt), paying a sure return $(1 + r)D$. The returns to equity after debt service will be $\bar{x} + \mu - (1 + r)D$, with value $V(\alpha) - D$. If the entrepreneur sells a proportion $(1 - \alpha)$ of his equity (retaining a proportion $\alpha$), he will receive $(1 - \alpha)[V(\alpha) - D]$. His initial wealth after transactions related to the project is

$$W_0 + D + (1 - \alpha) \left[ V(\alpha) - D \right] - K.$$ 

This will be divided between investments in the market (which cost $\beta V_M$) and his private holdings of the riskless asset (which cost $Y$). Thus, the entrepreneur's budget constraint satisfies the equation (2).
by his returns from investments in the project, market, and riskless security:

\[ \tilde{W}_1 = \alpha [\tilde{x} + \mu - (1 + r)D] + \beta \tilde{M} + (1 + r)Y. \]

Substituting for \( Y \) from (2) and for \( V(\alpha) \) from (1) yields

\[ \tilde{W}_1 = \alpha [\tilde{x} + \mu - \mu(\alpha) + \lambda] + \beta [\tilde{M} - (1 + r)V_M] + (W_0 - K)(1 + r) + \mu(\alpha) - \lambda. \tag{3} \]

The decision problem

\[ \text{Maximize } E[U(\tilde{W}_1)] \tag{4} \]

will, for any given \( \mu(\alpha) \) schedule, determine an optimal portfolio which depends upon \( \mu \):

\[ \alpha^* = \alpha^*(\mu), \]

\[ \beta^* = \beta^*(\mu); \]

where \( \alpha^* \) and \( \beta^* \) are the optimal holdings of the project and the market portfolio, respectively. 6

We are not interested in arbitrary functions \( \mu(\alpha) \); rather, we shall restrict our attention to schedules which have an equilibrium property. More precisely, we define an

**Equilibrium Valuation Schedule:** A market valuation schedule \( \mu(\alpha) \) is said to be an equilibrium valuation schedule if the entrepreneur’s true \( \mu \) is correctly identified by the market for all values of \( \mu \) for which the entrepreneur undertakes the project. That is,

\[ \mu[\alpha^*(\mu)] = \mu, \tag{5} \]

for all levels of \( \mu \) which induce the entrepreneur to undertake the project, given the schedule \( \mu(\alpha) \).

Condition (5) is a natural notion of equilibrium given competitive capital markets. If the imputed \( \mu(\alpha) \) were greater than the actual \( \mu \) of an entrepreneur retaining \( \alpha \), outside investors would on average receive less than the return required for the project’s risk, and equity financing would not continue on such terms. If, on the other hand, \( \mu(\alpha) \) consistently underestimated the entrepreneur’s true \( \mu \), given \( \alpha \), excess returns would exist for outside investors. Competitive forces would eliminate

6. Note \( D \) has disappeared from equation (3), and therefore from the maximization problem. Substituting the optimal holdings \( \alpha^* \) and \( \beta^* \) into the budget constraint (2) determines

\[ H^* \equiv \alpha^*D - Y \]

only. Any combination of borrowing \( D \) through the firm or \(-Y\) on personal account which satisfies the above equation will generate the same expected utility to the entrepreneur, if there are no transactions costs. Discussion of an optimal \( D \) is deferred until pp. 14 ff.
these excess returns. Thus, for levels of \( \mu \) for which entrepreneurs undertake their projects, (5) must hold in equilibrium.\(^7\)

**Properties of Equilibrium Valuation Schedules**

We shall not address the difficult problem of whether an equilibrium schedule \( \mu(\alpha) \) exists.\(^8\) Rather, we shall presume that at least one equilibrium schedule exists, and examine its properties. In the subsequent section, we consider an example in which we can actually compute an equilibrium valuation schedule.

Consider now any equilibrium schedule \( \mu(\alpha) \). Given this schedule and value of \( \mu \) in the relevant range, the entrepreneur chooses \( \alpha \) and \( \beta \) to maximize expected utility (4). First-order necessary conditions require that

\[
\frac{\partial E\left[U(\tilde{W}_i)\right]}{\partial \alpha} = E\left[U'(\tilde{W}_i)(\tilde{x} + \mu - \mu(\alpha) + \lambda + (1 - \alpha)\mu_{\alpha})\right] = 0; \tag{6}
\]

\[
\frac{\partial E\left[U(\tilde{W}_i)\right]}{\partial \beta} = E\left[U'(\tilde{W}_i)(\tilde{M} - (1 + r)V_M)\right] = 0; \tag{7}
\]

where \( \tilde{W}_i \) is given by (3) and \( \mu_{\alpha} \equiv d\mu(\alpha)/d\alpha \).

Note that condition (6) diverges from the usual portfolio optimization condition because the "price" of the project, \( V(\alpha) \), depends upon \( \alpha \) through the equilibrium valuation schedule \( \mu(\alpha) \). Using the equilibrium condition (5) and rearranging terms, we may rewrite (6) as

\[
(1 - \alpha)\mu_{\alpha} = -E\left[U'(\tilde{W}_i)(\tilde{x} + \lambda)\right]/E\left[U'(\tilde{W}_i)\right]. \tag{8}
\]

Equation (7) can now be used to solve for \( \beta \) as a function of \( \alpha \) and \( \mu \).

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7. Because condition (5) is key to the analysis which follows, it behooves us to make more precise the values of \( \mu \) for which projects will be undertaken, given an equilibrium schedule \( \mu(\alpha) \). Let us define the reservation utility level \( U^* \) as the maximum expected utility the entrepreneur could achieve without undertaking the project. Let \( \mu(\alpha) \) be an assumed equilibrium valuation function. Then condition (5) need hold only for values of \( \mu \) such that

\[
\max_{\alpha,\beta} E\left[U(\tilde{W}_i)\right] > U^*;
\]

Let \( \mu^* \) represent the minimum level of \( \mu \) for which the inequality holds. Clearly \( \mu^* \) depends on the schedule \( \mu(\alpha) \).

It can be shown that, for a given equilibrium valuation function \( \mu(\alpha) \), the project will be undertaken if and only if \( \mu \geq \mu^* \). Thus, the relevant range \( R[\mu(\alpha)] \) of the equilibrium schedule \( \mu(\alpha) \) is given by

\[
R[\mu(\alpha)] = \{\mu | \mu > \mu^*\}
\]

and the relevant domain \( D[\mu(\alpha)] \) by

\[
D[\mu(\alpha)] = \{a \in [0,1] | \mu(\alpha) > \mu^*\}.
\]

Note that, in equilibrium, we will never find more productive projects being rejected while less productive projects are undertaken.

8. Clearly this assumption is nontrivial. In the example considered in the subsequent section, however, a schedule exists which satisfies the equilibrium conditions. See also Riley [1976].
Substituting this relationship for $\beta$ into (8) yields a differential equation relating $\mu$ and $\alpha$. Any equilibrium schedule must satisfy this differential equation over the relevant domain.

In addition to the necessary first-order conditions, the entrepreneur's optimal choice of $\alpha$ and $\beta$ must satisfy second-order optimizing conditions. Defining

$$A \equiv \partial^2 E\left[ U(\tilde{W}_1) \right] / \partial \alpha^2 = E\left[ U''(\tilde{W}_1) \left[ \tilde{x} + \lambda + (1 - \alpha)\mu_\alpha \right]^2 \right]$$

$$+ E\left[ U'(\tilde{W}_1) \left[ -2\mu_\alpha + (1 - \alpha)\mu_\alpha \alpha \right] \right];$$

$$B \equiv \partial^2 E\left[ U(\tilde{W}_1) \right] / \partial \beta^2 = E\left[ U''(\tilde{W}_1) \left[ \tilde{M} - (1 + r)V_M \right]^2 \right];$$

$$C \equiv \partial^2 E\left[ U(\tilde{W}_1) \right] / \partial \alpha \partial \beta = E\left[ U''(\tilde{W}_1) \left[ \tilde{x} + \lambda + (1 - \alpha)\mu_\alpha \right] \left[ \tilde{M} - (1 + r)V_M \right] \right],$$

second-order conditions require

$$A < 0; \quad B < 0; \quad AB - C^2 > 0. \quad (9)$$

The necessary conditions (8) and (9) will be used to examine properties of equilibrium valuation schedules. But first, we need a definition:

**Normal Asset Demand:** An individual’s demand for an asset is said to be normal if, in a portfolio choice situation without signaling, the individual will always demand a larger amount of that asset when its price falls.

**Theorem I.** The equilibrium valuation function $\mu(\alpha)$ is strictly increasing with $\alpha$ over the relevant domain, if and only if the entrepreneur’s demand for equity in his project is normal.

**Proof.** See Appendix.

Theorem I provides a fairly strong characterization of equilibrium schedules: under normal conditions they are monotonically increasing with the fraction of ownership $\alpha$ retained by the entrepreneur. The market reads higher entrepreneurial ownership as a signal of a more favorable project. And the entrepreneur is motivated to choose a higher fraction of ownership in more favorable projects, given the equilibrium valuation function.

**Theorem II.** In equilibrium with signaling by $\alpha$, entrepreneurs with normal demands will make larger investments in their own projects than would be the case if they could costlessly communicate their true mean.

**Proof.** See Appendix.

Theorem II can be viewed as a welfare result: the “cost” of signaling the true $\mu$ to the market through $\alpha$ is the welfare loss resulting from investment in one’s own project beyond that which would be optimal if the true $\mu$ could be communicated costlessly. Of course, less costly communication may not be possible. And, as argued in the introduction, equilibrium with no communication could result in no projects being undertaken.
To examine further aspects of equilibrium valuation schedules and their implications for financial structure, we turn our attention to a specific example.

**The Signaling Model: An Example**

Let us assume

(a) Entrepreneur's expected utility can be expressed in the form

\[
E\left[ U(\tilde{W}_1) \right] = G\left[ E(\tilde{W}_1) - (b/2)\sigma^2(\tilde{W}_1) \right],
\]

where \( G \) is a monotonically increasing function and \( \sigma^2(\tilde{W}_1) \) is the variance of end-of-period wealth;\(^9\)

(b) The risk adjustment coefficient can be expressed as \( \lambda = \lambda^* \text{Cov}(\tilde{x}, \tilde{M}) \), where

\[
\lambda^* = \frac{E(\tilde{M}) - (1 + r)V_M}{\sigma_M^2} \tag{11}
\]

is the "market price of risk." Assumption (b) is consistent with the valuation implied by the capital asset pricing model.\(^10\)

Using (10), first-order maximizing conditions (6) and (7) can be expressed as

\[
\left[ \mu - \mu(a) + \lambda^* \text{Cov}(\tilde{x}, \tilde{M}) \right] + (1 - \alpha)\mu_a - ab\sigma^2 - \beta b \text{Cov}(\tilde{x}, \tilde{M}) = 0; \tag{12}
\]

\[
\left[ E(\tilde{M}) - (1 + r)V_M \right] - ab\text{Cov}(\tilde{x}, \tilde{M}) - \beta b\sigma_M^2 = 0, \tag{13}
\]

where \( \sigma^2_x \) and \( \sigma^2_M \) are the variance of the project and market returns, respectively. Substituting for \( \beta b \) from (13) and using (11) and (5) permits us to rewrite (12) as a special case of (8):

\[
(1 - \alpha)\mu_a = b\alpha\left[ Z \right], \tag{14}
\]

where

\[
Z = \frac{\sigma_x^2 \sigma_M^2 - \left[ \text{Cov}(\tilde{x}, \tilde{M}) \right]^2}{\sigma_M^2}. \]

Note that \( Z \) will always be nonnegative, and can be interpreted as the specific risk of the project. If the project is independent of the market returns, \( \text{Cov}(\tilde{x}, \tilde{M}) = 0 \) and \( Z \) is simply the variance of \( \tilde{x} \). If the market and project returns are perfectly correlated, \( Z = 0 \). In most cases, of course, \( Z \) will lie between these extremes.

The solution to the differential equation (14) is a family of functions

\[
\mu(a) = -bZ\left[ \log(1 - \alpha) + \alpha \right] + C \tag{15}
\]

9. Such a representation of expected utility is possible whenever indifference curves are linear in mean and variance. An example of this representation is when utility is exponential and returns are normally distributed.

10. See the models of Sharpe [1964] and Mossin [1966], for example.
where $C$ is an arbitrary constant. It can be verified that the second-order conditions (9) are also satisfied for schedules in the family.

Figure 1 shows some examples of valuation functions satisfying the equilibrium form (15). We will now show that further equilibrium arguments can be used to reduce this family of curves to a single schedule which will be viable in the market.

First, consider a curve above and to the left of $KK'$, such as $JJ'$. Because such a curve intersects the $\alpha=0$ axis at $\mu_j(0) > (1 + r)K + \lambda$, an entrepreneur could undertake a project with arbitrarily low true $\mu$, retain zero equity, and have

$$\max_{\beta} E \left[ U \left[ \beta \left( \tilde{M} - (1 + r) V_M \right) + (1 + r) \left( V_j(0) - K + W_0 \right) \right] \right]$$

$$> \max_{\beta} E \left[ U \left[ \beta \left( \tilde{M} - (1 + r) V_M \right) + W_0(1 + r) \right] \right],$$
since

\[ V_J(0) - K = \frac{1}{1+r} \left[ \mu_J(0) - \lambda \right] - K > 0. \]

Thus, at \( \alpha = 0 \), schedules such as \( JJ' \) have the property that entrepreneurs with true \( \mu < \mu_J(0) \) could undertake the project, retain zero equity, and be better off than they would if they abandoned the project. Lenders offering the schedule \( JJ' \) would lose money on projects in which the entrepreneur held zero equity. And indeed, even if lenders attach a minimum permissible \( \alpha > 0 \) to the schedule \( JJ' \), there will always be some “freeloaders” at \( \alpha \) whose \( \mu \)'s are less than those expected by the market. Schedules such as \( JJ' \) therefore will not satisfy the equilibrium condition (5) at their left endpoint.

If the schedule \( KK' \) is offered, the same freeloading problem as above may arise at \( \alpha = 0 \). (Since \( \alpha = 0 \) implies \( V_K(0) = K \), entrepreneurs with \( \mu < \mu_K(0) \) are indifferent between undertaking (holding zero equity) or not undertaking their projects.) But for \( \alpha > 0 \), no freeloading will take place, since even the smallest amount of required equity holding would reduce potential freeloaders to a level of utility less than that which would result if they did not undertake the project.

Now consider schedules below and to the right of \( KK' \), such as \( LL' \). (These schedules do not reach \( \alpha = 0 \) because the relevant domain does not include \( \alpha \)'s associated with levels of \( \mu \) less than \( \mu^* \).) Such schedules would indeed satisfy the equilibrium requirement (5). But they will not be competitive with schedule \( KK' \), in the sense that, if \( KK' \) were offered by some lenders when others were offering \( LL' \), all entrepreneurs would do business with lenders offering schedule \( KK' \). This is because the entrepreneur will have a higher level of expected utility along \( KK' \), since the required \( \alpha \) to signal any given level of \( \mu \) is less with \( KK' \) than with \( LL' \).

The cost of signaling is less along \( KK' \) than along any other schedule which satisfies the equilibrium requirement everywhere.\(^{11} \)

Thus, of all the family of functions (15), we have identified the unique equilibrium schedule

\[ \mu(\alpha) = -bZ[\log(1-\alpha) + \alpha] + (1+r)K + \lambda; \quad \alpha > 0. \tag{16} \]

Using (1),

\[ V(\alpha) = \frac{1}{1+r} \left[ -bZ[\log(1-\alpha) + \alpha] \right] + K, \quad V_a = \frac{bZ}{1+r} \left[ \frac{\alpha}{1+\alpha} \right] > 0, \]

implying

\[ V(0) = K; \quad V(\alpha) > K \quad \text{for} \quad \alpha > 0. \tag{17} \]

An immediate implication of (17) is

11. Riley [1975] argues along similar lines to reduce multiple schedules to a single schedule. If the number of projects offering a true value \( \mu \) or more falls rapidly as \( \mu \) increases, we need not be concerned by the unravelling problem considered by Rothschild and Stiglitz [1975] and by Riley [1975; amended 1976]. Even the left endpoint is immune to unravelling since there is by assumption a number of projects whose true \( \mu \) lies below cost \( K \).
PROPOSITION I. *A project will be undertaken if, and only if, its true market value, given \( \mu \), exceed its cost.*\(^{12}\)

This result implies that information transfer through signaling possesses a key *efficiency property*: the set of projects which are undertaken will coincide with that set which would be undertaken if information could be communicated costlessly.\(^{13}\)

We now consider the effects of parametric changes on the signaling equilibrium.

**PROPOSITION II.** *An increase either in the specific risk \( Z \) of the project or in the risk aversion \( b \) of the entrepreneur will reduce the entrepreneur’s equilibrium equity position \( \alpha^*(\mu) \), for any value of \( \mu \) at which the project is undertaken.*

**Proof.** For any fixed value of \( \mu \), we have from the equilibrium requirement \( \mu(\alpha) = \mu \)

\[
\frac{d}{d(bZ)} \left[ \mu(\alpha) \right] = 0; \text{ or }
\]

\[
\frac{d\alpha}{d(bZ)} \left[ -bZ[\log(1-\alpha) + \alpha] + (1+r)K+\lambda \right] = 0;
\]

implying

\[
\frac{d\alpha}{d(bZ)} = \frac{(1-\alpha)[\log(1-\alpha) + \alpha]}{abZ} < 0,
\]

since

\[
[\log(1-\alpha) + \alpha] < 0 \quad \text{for} \quad \alpha > 0.
\]

A more fundamental question concerns entrepreneurial welfare: does the expected utility “cost” of signaling vary with \( Z \)? We can show

**PROPOSITION III.** *An increase in specific risk \( Z \) results in greater expected utility for the entrepreneur, for any level \( \mu \) at which the project is undertaken.*

**Proof.** Tiedious calculations show that

\[
\frac{dE[U]}{dZ} = G'(\gamma) \left[ -b(1-\alpha)[\log(1-\alpha) + \alpha] \right] > 0
\]

Thus projects which are “more distinct” (higher specific risk) from the market are relatively easier to signal, in the sense that they result in lower signaling costs in equilibrium.

12. This result can be generalized to arbitrary utility functions, only if (as in this example) the optimal holding of the project is zero when \( \mu \) is directly communicated. Such will be the case if the entrepreneur (like other investors) wishes to hold “the market portfolio” in the absence of signaling.

13. This is not quite correct: since we require \( \alpha > 0 \), only projects with \( \mu > K \) will be undertaken. With costless information transfer, projects with \( \mu = K \) might be undertaken, since entrepreneurs would be indifferent between acceptance or rejection of these projects. But since projects with \( \mu = K \) are truly marginal to society, there is no social loss associated with their exclusions. Note that, as \( \alpha \to 0 \) and \( \mu \to K \), the expected utility loss associated with signaling approaches zero. This explains why the acceptance sets are identical.
OPTIMAL DEBT LEVELS AND THE MODIGLIANI-MILLER THEOREM

We have shown that in equilibrium the entrepreneur's equity position \( \alpha \) in his project is related to the value of his project. We now address the relationship between the value of the project (or firm) \( V \) and the financing decision \( D \). In a world of symmetric information, the Modigliani-Miller theorem suggests that there will be no systematic relationship between the financing decision and the value of the firm. In a world with asymmetric information, we show that this will not always be the case. But our results must be interpreted with considerable caution.

From the budget constraint (2), we observe

\[
H \equiv \alpha D - Y = K - W_0 - (1 - \alpha)V(\alpha) + \beta V_M. \tag{18}
\]

For the subsequent discussion, we consider the example introduced in the previous section, with the additional assumption that the project's returns are independent of the market returns. This implies \( \text{cov}(\lambda, M) = 0 \), which in turn can be shown to imply that \( \beta \) is independent of \( \alpha \). Thus we can talk of \( Z \) as the variance of the project's returns, and can treat \( \beta V_M \) as a constant with respect to the choice \( \alpha \).

We shall make the assumption that (as both debt and lending are at the riskless rate) the entrepreneur will not simultaneously borrow and lend: borrowing will be done through the firm, and lending will be done privately. Institutional arrangements and (even small) transactions costs can be invoked to support the realism of this argument.

We focus upon the domain of \( \alpha \) where the debt of the firm is positive. In this domain, \( Y = 0 \), and assuming for simplicity \( r = 0 \), we can rewrite (18) as

\[
D = \left[ K - W_0 + \beta V_M - (1 - \alpha)V(\alpha) \right] / \alpha \\
= \left[ K - W_0 + \beta V_M - (1 - \alpha)[-bZ[\log(1 - \alpha) + \alpha] + K] \right] / \alpha, \tag{19}
\]

using (16). Differentiating with respect to \( \alpha \) gives

\[
\text{sign}(\partial D / \partial \alpha) = \text{sign} \left\{ -bZ[\log(1 - \alpha) + \alpha + \alpha^2] + W_0 - \beta V_M \right\}. \tag{20}
\]

\( \partial D / \partial \alpha \) will not be positive for all values of \( \alpha \), \( W_0 \), and \( bZ \). Calculations show, however, that if

\[
\frac{W_0 - \beta V_M}{K} > .186
\]

then \( \partial D / \partial \alpha > 0 \) for all levels of \( \alpha \) such that \( D > 0 \). This is true regardless of the absolute levels of \( K \) and \( W_0 \), as well as the level of \( bZ \). Therefore debt will be an increasing function of the entrepreneur's equity position \( \alpha \), whenever the entrepreneur's financial contribution to the firm is at least 18.6 percent of the cost of the project.\(^{14} \) We shall assume this to be the case, and henceforth limit our attention to debt which is monotonically increasing in \( \alpha \).

\(^{14} \) In most cases, \( \partial D / \partial \alpha > 0 \) even when entrepreneurs own considerably less than 18.6 percent of the firm. But this fraction ownership guarantees under all circumstances that debt increases with equity share, when debt is positive.
**Proposition IV.** For any level of \( \mu \), greater project variance \( \sigma_x^2 \) implies lower optimal debt.

**Proof.** Differentiating (19) with respect to \( Z(=\sigma_x^2) \), keeping \( \mu \) constant, yields

\[
\frac{dD}{dZ} = \left( \frac{\partial D}{\partial \alpha} \right) \left( \frac{\partial \alpha}{dZ} \right) + \frac{\partial D}{\partial Z} \\
= \left( \frac{\partial D}{\partial \alpha} \right) \left( \frac{\partial \alpha}{dZ} \right) + \left[ (1-\alpha) b \left( \log(1-\alpha) + \alpha \right) \right] / \alpha.
\]

By our previous analysis, \( \partial D/\partial \alpha > 0 \); by Proposition II, \( \partial \alpha/dZ < 0 \). Since \( \log(1-\alpha) + \alpha < 0 \) for all \( \alpha > 0 \), it follows that \( dD/dZ < 0 \).

Proposition IV shows that, independent of possible bankruptcy costs, firms with riskier returns will have lower optimal debt levels.\(^{15}\)

Consider now the relationship between value \( V \) and debt \( D \) of seemingly similar projects. By “seemingly similar,” we mean that observers without inside information on \( \mu \) view the projects as identical. Since both \( V \) and \( D \) are positive functions of \( \alpha \), and therefore of \( \mu \), a regression of value on debt would show a positive relationship.

Does this invalidate the Modigliani-Miller theorem that value is independent of capital structure? Not really. In the MM world with symmetric information, a change in \( D \) will not change the project’s perceived returns, and financial structure will be irrelevant. In a world with asymmetric information in which \( \alpha \) can be observed, a change in \( D \) with \( \alpha \) constant will not change perceived returns, and financial structure will also be irrelevant. But we have argued that observed \( D \), given small transactions costs, will be related to \( \alpha \). And a change in \( \alpha \) does give rise to a change in perceived returns and therefore in market value. Thus there is a statistical but not a causal relation between \( V \) and \( D \) of seemingly similar firms.

If transactions costs were sufficiently high, or institutions such that borrowing through the firm entirely precluded lending privately, then \( D \) itself could serve as a signal of \( \mu \) and therefore of firm value, since a choice of \( D \) would (through the budget constraint) determine a unique choice of \( \alpha \); \( D \) as well as \( \alpha \) would then be a function of \( \mu \) and could serve as a signal. But when transactions costs are minimal, \( D \) cannot serve as a signal, since entrepreneurs with any \( \mu \) would be willing to incur small transactions costs to have (say) high \( D \)’s in order to receive a high project value, while at the same time choosing \( Y \) so that \( \alpha \) remained at a level appropriate to their true \( \mu \). Thus \( D \) could not serve as a signal with equilibrium properties.

**Information and Financial Intermediation: Some Preliminary Thoughts**

Traditional models of financial markets have difficulty explaining the existence of financial intermediaries, firms which hold one class of securities and sell securities of other types. If transactions costs are not present, ultimate lenders might just as well purchase the primary securities directly and avoid the costs which intermediation must involve. Transactions costs could explain intermediation, but their

\(^{15}\) Note that this result would follow in a normal portfolio model (without signaling), if we assumed as here that transactions costs linked debt to the entrepreneur’s choice of \( \alpha \), and further assumed that optimal holding \( \alpha \) increased with \( \mu \).
magnitude does not in many cases appear sufficient to be the sole cause. We suggest that informational asymmetries may be a primary reason that intermediaries exist.

For certain classes of asset—typically, those related to individuals, such as mortgages or insurance—information which is not publicly available can be obtained with an expenditure of resources.\(^\text{16}\) This information can benefit potential lenders; if there are some economies of scale, one might expect organizations to exist which gather and sell information about particular classes of assets.

Two problems, however, hamper firms which might try to sell information directly to investors. The first is the appropriability of returns by the firm—the well known “public good” aspect of information. Purchasers of information may be able to share or resell their information to others, without diminishing its usefulness to themselves.\(^\text{17}\) The firm may be able to appropriate only a fraction of what buyers in totality would be willing to pay.

The second problem in selling information is related to the credibility of that information. It may be difficult or impossible for potential users to distinguish good information from bad. If so, the price of information will reflect its average quality. And this can lead to market failure, if entry is easy for firms offering poor quality information. Firms which expend considerable resources to collect good information will lose money because they will receive a value reflecting the low average quality. When they leave the market, the average quality will further fall, and equilibrium will be consistent only with poor quality information, much as Akerlof’s market for used cars will result in only “lemons” for sale.

Both these problems in capturing a return to information can be overcome if the firm gathering the information becomes an intermediary, buying and holding assets on the basis of its specialized information. The problem of appropriability will be solved because the firm’s information is embodied in a private good, the returns from its portfolio. While information alone can be resold without diminishing its returns to the reseller, claims to the intermediary’s assets cannot be. Thus, a return to the firm’s information gathering can be captured through the increased value (over cost) of its portfolio.\(^\text{18}\)

Of course, a return to information can be gathered only if the buyers of the intermediary’s claims believe that the intermediary uses good information. Without

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16. We do not consider information of this type to be a normal “transactions cost.” Securities can be exchanged in many cases independently of how much the buyer may know about the precise nature of that security. Costs of exchange are considered transactions costs. We do not consider better information or “sorting costs” to be a cost necessary for exchange. Our approach can be contrasted with that of Benston [1976]. By emphasizing informational asymmetries, we can show not only why financial intermediaries exist, but also why they tend to be characterized by high leverage.

17. This, of course, depends on the nature of the information. If information provides a competitive advantage, its usefulness may be diminished by resale. If it simply provides better decisions internally, its usefulness may not be.

18. We presume that outsiders cannot profit from the firm’s information merely by examining the portfolio chosen by the firm. Note that some loss in the potential value of information may occur because the portfolio weights chosen by the firm may not coincide with the weights which individual investors would have chosen by themselves, given the information. If there is competition for assets which have been sorted, firms may have difficulty in achieving a return to their information unless the borrowers directly pay for their sorting: see Stiglitz [1974].
some signal of quality, the average return may be low. But, just as in previous sections, this problem can be overcome through signaling. The organizers' willingness to invest in their firm's equity serves as a signal of the quality of the firm's information and the assets selected on the basis of this information. We previously have shown that the financial structure of the firm—the types and amounts of securities it issues—will be related to the owner's equity share. If, as seems often the case, most intermediaries' assets have low specific risk, Proposition IV implies the high degrees of leverage (through debt or deposits) which characterize most intermediaries.

It is of interest to note that, once an organization or group of organizations becomes more capable than other lenders of sorting a class of risks, there is a natural tendency for such assets to be sorted—even when the information costs of doing the sorting may be relatively high. Sellers of risks with favorable characteristics wish to be identified, and would deal with an informationally-efficient intermediary rather than with an uninformed set of lenders offering the value of the average risk. With the best risks "peeled off," the average risk will be less valuable, inducing owners of the next best risks to deal with the intermediary. The end of this chain of logic is that sellers of all types of risks will sell to the intermediary, except perhaps the group at the bottom of the barrel. An open question is whether, in equilibrium, an optimal amount of sorting occurs.

REFERENCES


APPENDIX

Proof of Theorem 1

We first shall show that normality implies $\mu_\alpha$ is positive over the relevant domain. Differentiating the first-order conditions (6) and (7) totally with respect to $\mu$ and
using the equilibrium condition (5) gives

\[
\begin{bmatrix}
A & C \\
C & B \\
\end{bmatrix}
\begin{bmatrix}
\frac{d\alpha^*}{d\mu} \\
\frac{d\beta^*}{d\mu} \\
\end{bmatrix} = -\begin{bmatrix}
D \\
G \\
\end{bmatrix},
\]

where \(A, B,\) and \(C\) are given on page 8, and

\[
D \equiv E\left[U'(\tilde{W}_1) + E\left[U''(\tilde{W}_1)\left[\tilde{x} + \lambda + (1 - \alpha)\mu_\alpha\right]\right]\right],
\]

\[
G \equiv E\left[U''(\tilde{W}_1)\left[M - (1 + r) V_M\right]\alpha\right],
\]

evaluated at \(\alpha = \alpha^*\). Solving for the unknowns gives

\[
\begin{bmatrix}
\frac{d\alpha^*}{d\mu} \\
\frac{d\beta^*}{d\mu} \\
\end{bmatrix} = \left(\frac{1}{AB - C^2}\right)
\begin{bmatrix}
B & -C \\
-C & A \\
\end{bmatrix}
\begin{bmatrix}
-D \\
-G \\
\end{bmatrix}
\]

(A.2)

or

\[
\frac{d\alpha^*}{d\mu} = -R\begin{bmatrix}
DB - CG \\
\end{bmatrix},
\]

(A.3)

where \(R = 1/(AB - C^2) > 0\) by (9).

\(R > 0\) is required for a regular local maximum of expected utility. For a set of measure zero in which the second-order condition vanished (but higher-order conditions were satisfied), we would have \(\mu_\alpha = 0\).

Differentiating (5), we see that \((\mu_\alpha)(d\alpha^*/d\mu) = 1\); therefore,

\[
\text{sign} \mu_\alpha = \text{sign}(d\alpha^*/d\mu).
\]

(A.4)

We shall now show that \((d\alpha^*/d\mu)\) is positive by showing \(DB - CG\) is negative. Consider the regular portfolio problem

\[
\max_{\alpha, \beta} E\left[U(\tilde{W}_1)\right]
\]

subject to the budget constraint

\[
\alpha p_x + \beta p_M + Y = W_0,
\]

(A.5)

where \(\alpha\) and \(\beta\) are holdings of the project and market portfolio (with returns \(\tilde{x} + \mu\) and \(\tilde{M}\), as before), and \(p_x\) and \(p_M\) are arbitrary prices of these assets.

Final wealth \(\tilde{W}_1\) will, after substitution for \(Y\) through (A.5), be given by

\[
\tilde{W}_1 = \alpha\left[\tilde{x} + \mu - (1 + r)p_x\right] + \beta\left[\tilde{M} - (1 + r)p_M\right] + W_0(1 + r),
\]

and first-order maximizing conditions by

\[
\begin{align*}
E\left[U'(\tilde{W}_1)\left[\tilde{x} + \mu - (1 + r)p_x\right]\right] &= 0; \\
E\left[U'(\tilde{W}_1)\left[\tilde{M} - (1 + r)V_M\right]\right] &= 0.
\end{align*}
\]

(A.6)
Consider now the optimal choice of $\alpha$ and $\beta$ in this setting if

\[
\begin{align*}
    p_x &= \frac{1}{1+r} \left[ \mu(\alpha^*) - \lambda - (1-\alpha^*)\mu_a(\alpha^*) \right] \\
    p_m &= V_M \\
    W_0 &= (W_0 - K) + \frac{1}{1+ r} \left[ \mu(\alpha^*) - \lambda + (1-\alpha^*)\mu_a(\alpha^*) \right] \alpha^*,
\end{align*}
\]

(A.7)

where $\alpha^*$ is the optimal $\alpha$ chosen by the entrepreneur (given $\mu$) in the signaling environment. Substitution of these values into the first order conditions (A.6) yield precisely the conditions (6) and (7) when (5) holds; therefore, the same $\alpha^*, \beta^*$ \textit{will be chosen in this regular portfolio problem}, given the prices and wealth specified by (A.7).

Finally, consider a change in the price $p_x$ from that specified in (A.7). Differentiating (A.6) totally with respect to $p_x$, we find

\[
\begin{bmatrix}
    \hat{A} & C \\
    C & B
\end{bmatrix}
\begin{bmatrix}
    d\alpha / dp_x \\
    d\beta / dp_x
\end{bmatrix} =
\begin{bmatrix}
    (1+r)D \\
    (1+r)G
\end{bmatrix},
\]

where $B$ and $C$ are as given on page 8, $D$ and $G$ are given by (A.1), and

\[
\hat{A} = E \left[ U''(\hat{W}_1) (\hat{x} + \lambda + (1-\alpha^*)\mu_a^*)^2 \right].
\]

Solving for $d\alpha / dp_x$ gives (after matrix inversion)

\[
d\alpha / dp_x = R(1+r) \left[ DB - CG \right],
\]

where $R = \hat{A}B - C^2 > 0$ by the regular second-order portfolio conditions. But by the assumption of normality, $d\alpha / dp_x < 0$. Therefore,

\[
DB - CG < 0,
\]

which with (A.3) and (A.4) implies $\mu_a > 0$.

Proof of Theorem II

Consider $\alpha(k)$ and $\beta(k)$, the holdings of the project and the riskless asset for given $\mu$ which satisfy

\[
\begin{align*}
    E \left[ U'(\hat{W}_1)(\hat{x} + \lambda) \right] &= k; \quad \text{(A.8)} \\
    E \left[ U'(\hat{W}_1) \left( \hat{M} - (1+r) V_M \right) \right] &= 0,
\end{align*}
\]

when the value of $\mu$ is known to the public (costlessly) and does not depend upon $\alpha$. We note two special cases:

a) $k = 0$. In this case, the resulting solution to (A.8), $\alpha(0)$, is the optimal holding of the project by the entrepreneur if he could communicate $\mu$ costlessly to the public.
b) \( k = -(1 - \alpha^*) \mu^*_a E[U'(W^*_1)] \) where \( \alpha^* = \alpha^*(\mu) \), the optimal holding of the project when the market perceives \( \mu \) through the equilibrium schedule \( \mu(\alpha) \), and \( EU'(W^*_1) \) is expected utility when \( \alpha = \alpha^* \), \( \beta = \beta^* \). The solution \( \alpha(k) \) is simply \( \alpha^* \), since (A.8) in this case coincides with the conditions (6) and (7) when (5) holds.

What we shall now show is that \( \frac{d\alpha(k)}{dk} < 0 \), for any \( \mu \). Totally differentiating the conditions (A.8), we have

\[
\begin{bmatrix}
M & N \\
N & P
\end{bmatrix}
\begin{bmatrix}
\frac{d\alpha}{dk} \\
\frac{d\beta}{dk}
\end{bmatrix}
= \begin{bmatrix}
1 \\
0
\end{bmatrix},
\]

(A.9)

where \( M \equiv E[U''(W_1)(x+\lambda)^2] < 0; \quad N \equiv E[U''(W_1)(x+\lambda)[M-(1+r)V_M]] ; \quad \) and \( P \equiv E[U''(W_1)[M-(1+r)V_M]^2] < 0. \) By the concavity of \( U, MP - N^2 > 0 \), for all possible \((\alpha, \beta)\). Solving (A.9) for \( \frac{d\alpha}{dk} \) yields

\[
\frac{d\alpha}{dk} = \frac{1}{MP - N^2}(P) < 0, \quad \text{which was to be shown.}
\]

We finally observe that, in going from costless communication of \( \mu \) to signaling, the relevant first-order conditions go from \( k = 0 \) to \( k = -(1 - \alpha^*) \mu^*_a E[U'(W^*_1)] < 0 \), by Theorem I. Since \( \frac{d\alpha}{dk} < 0 \), \( \alpha \) will be larger with signalling.